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PROBLEMS WITH LOGICAL FORM

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0. Introduction. Problem formulation

The examination of quantifiers plays an essential role in modern linguistic theories. One of the most important issues in this respect was raised by Jaakko Hintikka (1973: 350), who proposed the following thesis:

(H) Certain natural language sentences require essential non-linear quantification¹ to adequately express their logical form.

In order to prove this kind of thesis one needs to provide examples and show that their adequate logical form can be expressed in elementary logic. Hintikka (1973: 344) believes the following sentence to be the simplest example in terms of syntax:

(1) Some relative of each villager and some relative of each townsman hate each other.²

The thesis proposed by Hintikka has sparked lively controversy (cf. Gabbay and Moravcsik 1974, Guenther and Hoepelman 1976, Stenius 1976, Hintikka 1976, Mostowski, Wojtyniak 2004). The two articles most relevant for this discussion, Barwise 1979 and Mostowski 1994, attempt to order and summarise the results of the discussions.³

¹Non-linear (branching) quantifiers have been introduced by Henkin (1961).

²It is assumed here that the sets of villagers and townsmen are mutually exclusive and that each townsman and each villager is his own relative.

³The author's most recent paper on the topic (Gierasimczuk and Szymanik 2009) contains a more elaborate discussion additionally supported by experimental evidence.

This paper is devoted to the logical form of sentence (1), which I will also refer to as the Hintikka sentence. Although the analysis presented below leads to the conclusion that nothing can determine the nonlinear nature of sentence (1), I am not attempting to undermine the (H) thesis. Despite considering sentence (1) as an unconvincing example, I believe that better examples have been provided in the debate. A particularly interesting example was formulated by Barwise (1979: 60):

(2) Most relatives of each villager and most relatives of each townsman hate each other.

In other words, the subject of a critical analysis will be the following thesis:

(H') In order to adequately express the logical form of (1) it is necessary to use non-linear quantifiers.

Below I will discuss the arguments formulated to support (H') and I will explain why I consider them insufficient.

1. Hintikka's arguments

Hintikka's reasoning was as follows. The phrase "some relative of each villager..." should have the following logical form: $\forall x \exists y [V(x) \rightarrow (R(x,y) \wedge \dots)]$. Similarly, the phrase "some relative of each townsman" should have the following form: $\forall z \exists w [T(z) \rightarrow (R(z,w) \wedge \dots)]$. If we join these two sequences in the following way:

(3) $\forall x \exists y \forall z \exists w [(V(x) \wedge T(z)) \rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))]$

(For each x , there exists a y and for each z there exists a w , such that if x is a villager, and z is a townsman, then y is a relative of x , w is a relative of z , and y and w hate each other.)

the choice of the relative of villager, y , will depend only on villager x , while the choice of the relative of townsman, w , will be determined both by villager x and by townsman z , which is clearly illustrated by the translation of this sentence into second-order language, where f and g are Skolem functions:

(4) $\exists f \exists g \forall x \forall z [(V(x) \wedge T(z)) \rightarrow (R(x,f(x)) \wedge R(z,g(x,z)) \wedge H(f(x),g(x,z)))]$

(There exist functions f and g , such that for each x and z , if x is a villager, and z is a townsman, then $f(x)$ is a relative of x , $g(x,z)$ is a relative of z , and $f(x)$ and

$g(x, z)$ hate each other.)

However, this interpretation of sentence (1) cannot be considered valid, as it suggests that (1) is not true in the same situations as the equivalent sentence:

(5) Some relative of each townsman and some relative of each villager hate each other.

For if we proceeded analogously to (1), we would assign the following logical form to (5):

$$(6) \forall z \exists w \forall x \exists y [(V(x) \wedge T(z)) \rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))]$$

which is equivalent to:

$$(7) \exists g \exists f \forall z \forall x [(V(x) \wedge T(z)) \rightarrow (R(x, f(z, x)) \wedge R(z, g(z)) \wedge H(f(z, x), g(z)))]$$

Now the choice of the relative of townsman, w , depends only on townsman z , and the choice of the relative of villager, y , depends both on townsman z and on villager x . Hintikka claims that the linear-quantifier reading of (1) and (5) is inconsistent with the fact that both sentences have identical truth conditions. However, (3) is not equivalent to (6). Hintikka concludes that (3) is not an adequate logical form of (1). Up to this point, I agree with the Finnish philosopher.

Yet Hintikka goes further and claims that consequently we need a formula in which neither the prefix " $\forall x \exists y$ " precedes " $\forall z \exists w$ " nor the other way round, and proposes to ascribe the following logical form to sentence (1):

$$(8) \quad \forall x \exists y^4$$

$$[(V(x) \wedge T(z)) \rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))]$$

$$\forall z \exists w$$

(For each x , there exists a y , and independently, for each z there exists a w , such that...)

which takes the following form after applying the Skolem function:

⁴In this formula, each existential variable depends on all the universal variables that occur earlier in the same branch and only on them.

$$(9) \exists f \exists g \forall x \forall z [(V(x) \wedge T(z)) \rightarrow (R(x, f(x)) \wedge R(z, g(z)) \wedge H(f(x), g(z)))].$$

(There exist functions f and g , such that for each x and z , if x is a villager and z is a townsman, then $f(x)$ is a relative of x , $g(z)$ is a relative of z , and $f(x)$ and $g(z)$ hate each other.)

In other words, the choice of the relative of villager, y , depends only on villager x , and the choice of the relative of townsman, w , depends only on townsman z . The logical form of (8) meets the condition of equivalence of sentences (1) and (5) (Hintikka 1973: 345). Formula (8) is not equivalent to any formula of elementary logic (Barwise 1979: 71) and thus we would say that it is essentially non-linear. I will refer to (8) as a 'strong reading' of (1).

But is Hintikka's reasoning a sufficient argument for (H')? Absolutely not. Hintikka tries to convince us that formula (3) is most certainly not the logical form of sentence (1) because (1) and (5) should have the same truth conditions. However, he does not take into account any alternative formula for (8), although there exist elementary logic formulae which could serve as the logical form of both (1) and (5), for instance the following:

$$(10) \forall x \forall z \exists y \exists w [(V(x) \wedge T(z)) \rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))]$$

Formula (10) represents a 'weak reading' of sentences (1) and (5).

We might want to ask why Hintikka failed to notice this possibility. Barwise suggests that this is due to the fact that (10) 'violates' the syntax of (1) by the unnatural (ad hoc) juxtaposition of the two "some of each" phrases (Barwise 1979: 53). However, this argument is unjustified, since the logical form — which we treat as the deep structure of the sentence — is almost always characterised by this kind of syntactic 'unnaturalness'. Let us consider for example the following sentences and their logical forms:

(11) Each human is mortal.

$$(12) \forall x [H(x) \rightarrow M(x)]$$

(For each x , if x is human, x is mortal.)

(13) 15ptAll books of a certain philosopher are worthless.

$$(14) \exists x \forall y [P(x) \wedge B(y) \wedge A(x, y) \rightarrow W(y)]$$

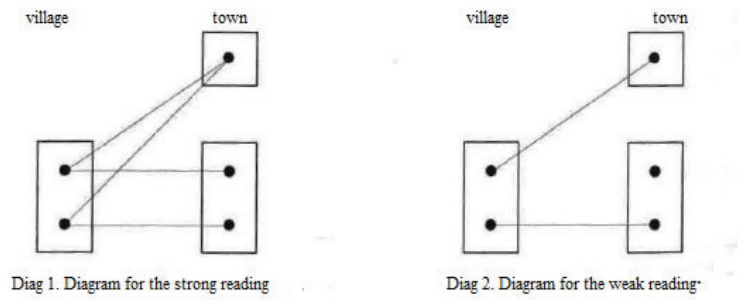
(There exist such x , that for each y , if x is a philosopher, and y is a book, and x

is the author of y , y is worthless.)

The logical forms ascribed to the above sentences do not seem controversial, although each of them violates the syntax in some way. In (11) there is no logical operator "if, then", while in (12) there is " \rightarrow ". In (13) the universal quantifier precedes the existential quantifier, but in (14) it is the other way round.

This illustrates the pointlessness of discussing the 'naturalness' of the logical form of a sentence. The most obvious criterion of adequacy of a logical form here is the conformity of truth conditions, i.e. we would say that the logical form of a natural language sentence is adequate if it is true only in the models in which this sentence is true. Thus, the problem analysed in this paper takes the form of the following question: are the truth conditions of sentence (1) reflected by formula (8) or (10)? These formulas are not equivalent; (8) implies (10), but there is no reverse implication.

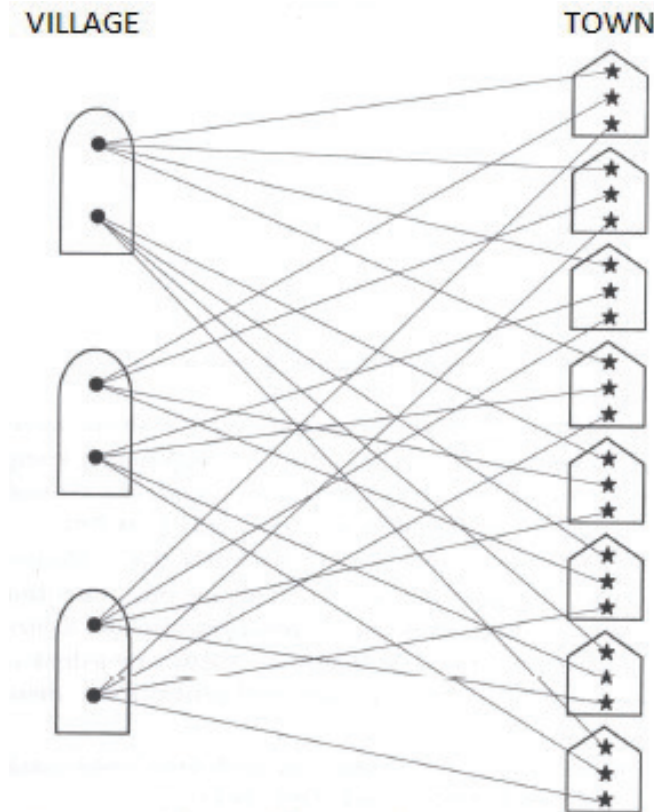
This relation can be illustrated by diagrams, with the village on the left and the town on the right; the dots within a given rectangle symbolise relatives, while the lines mean the relation of hatred.



2. Barwise's test

Barwise (1979) presents two arguments for assigning a linear logical form⁵ to sentence (1). The first is based on an empirical test of perceiving sentence (1) in a diagram. Barwise analyses a diagram in which the relations between townsmen and villagers are in a terrible state: every townsman hates every villager, except one, the one to whom he is connected by a line in Figure 3. In total, out of 144 (6×24) pairs of townsmen and villagers in Figure 3, 120 pairs hate each other, and only 24 do not hate each other. In the experiment, the subjects were asked the following question: in this diagram, is it or is it not the case that some relative of each villager and some relative of each townsman hate each other? In other words, is it or is it not the case that some dot in each hut and some star in each house are not connected by a line? I encourage the reader to try to answer this question.

⁵i.e. a logical form with a linear quantifier.

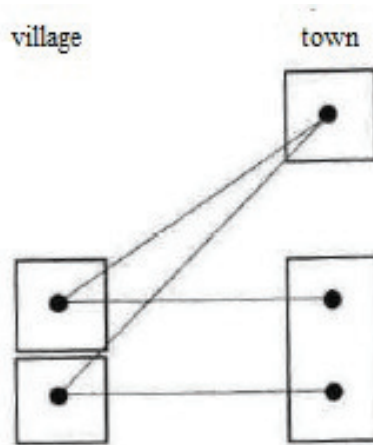


Diag. 3. Barwise's diagram

The reader who agrees that sentence (1) is true in the above illustration is rejecting the strong reading. The branching reading asserts that we can choose one villager (dot) out of each hut, once and for all, and one townsman (star) out of each house, again once and for all, and choose their relatives, and as a result the selected relatives (three dots and eight stars) will hate each other (will not be connected with a line). This is obviously impossible, as the readers may see for themselves. Barwise states: "In our experience, there is almost universal agreement that some dot in each hut and some star in each house are not connected by a line." (Barwise 1979: 51). Thus, Barwise's experiments⁶ indicate that the language users would consider the weak reading as referentially true.

The first doubt that comes to mind regarding the experiment is whether the graphical complexity of the diagram might have influenced the result. This issue was raised by Mostowski, who later proposed a modified diagram. In Figure 4, lines signify the relations of hatred.

⁶Barwise's experiment aims at determining whether language users would consider the Hintikka sentence true in the situation presented in Figure 3.



Diag 4. Simplified Barwise's diagram

Mostowski also states that this significant simplification rather does not affect the answers of the subjects (Mostowski 1994: 223). He suggests that the complexity experienced when examining Figures 3 and 4 may be caused by high algorithmical complexity of the problem itself (Mostowski 1994: 229).⁷

We should stress that both Barwise's and Mostowski's tests were only pilot questionnaires. Such experiments have never been conducted on a broader scale and the results have never been analysed by statistical methods. The attempts seem promising. There is, however, the essential question of how this type of research should be conducted in order for the results to be acceptable to us. What should the method of empirical experiments on the interpretation of certain sentences by language users look like, taking into account the computational complexity of the problems?⁸ Can such experiments be conclusive? Is statistical data on how people understand some sentences valid for research of the logical form? These questions require a separate paper, which should be devoted in large part to the notion of 'logical form' and the criteria of its adequacy.⁹

3. Inferential relations

⁷Using as their starting point the observation that a natural area of interpretation of the Hintikka sentence is a finite universe, Mostowski and Wojtyniak argue that the problem whether the strong reading of the Hintikka sentence is true in finite models is an NP-complete problem (Mostowski and Wojtyniak 2002: 6).

⁸A general introduction to computational complexity can be found in, e.g., (Papadimitriou 1993). A discussion on the relation between branching quantifiers and computational complexity can be found in (Blass and Gurevich 1986).

⁹For a paper attempting to settle those issues, see Gierasimczuk and Szymanik 2009.

Mostowski points out (Mostowski 1994: 219) that from (1) we are inclined to infer that:

(15) Each villager has a relative.

This observation is an argument for introducing a modification to the considered logical forms. Indeed, it does not follow from (8) and (10) that:

(16) $\forall x[V(x) \rightarrow \exists yR(x,y)]$

(For each x , if x is a villager, there exists a y who is a relative of x .)

while it is already implied by the corrected formulae (Mostowski 1994: 219—222). The strong version takes the form:

(17) $(\forall x:V(x))(\exists y:R(x,y))$

[H(y,w)]

$(\forall z:T(z))(\exists w:R(z,w))$

(For each x who is a villager there exists a y who is a relative of x , and, independently, for each z who is a townsman there exists a w who is a relative of z , such that y and w hate each other.)

An analogous correction for the weak version results in the following formula:

(18) $\forall x(V(x) \rightarrow \exists yR(x,y)) \wedge \forall z(T(z) \rightarrow \exists wR(z,w)) \wedge \forall x \forall z \exists y \exists w ((V(x) \wedge T(z)) \rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w)))$

(For each x , if x is a villager, there exists a y who is a relative of x , and for each z , if z is a townsman, there exists a w who is a relative of z , and, additionally, for any x, y, z, w , if x is a villager and z is a townsman, then y is a relative of x , w is a relative of z , and y and w hate each other.)

Instead of strengthening the logical form of (8) and (10) to (17) and (18) respectively, we can maintain that the logical form of (1) is (10) or (8), and the tendency to infer (15) from (1) can be explained by pragmatics — namely, by the fact that we are used to infer conclusions enthymematically from sentences on the basis of our entire knowledge — in this case, of the knowledge that the predicates "villager" and "townsman" are not empty.

The fact that (15) is a consequence of (1) is, however, not an argument for either the strong nor the weak reading of the Hintikka sentence, since in this context both forms (branched and linear) behave in the same way. Before the correction, there was no implication, whereas both (8) and (10) can easily be corrected to ensure this implication.

When referring to the inferential characteristics of (1), Mostowski challenges the weak reading in yet another way. From (1) and the sentence:

(19) John is a villager.

a competent language user will draw the following conclusion:

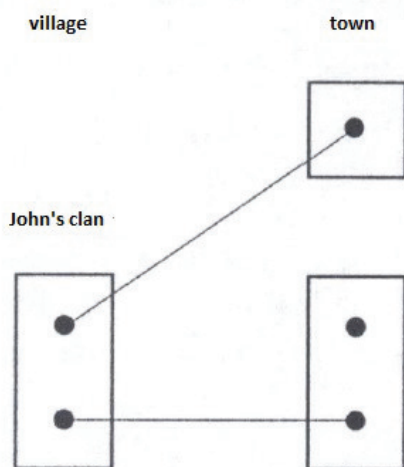
(20) Some relative of John and some relative of each townsman hate each other.

If we adopt the weak reading of (1), we will consider that (20) is true in Figure 5 presented below. Mostowski implicitly ascribes to sentence (20) the following logical form:

(21) $\exists x[R(\text{John},x) \wedge \forall y(T(y) \rightarrow \exists z(R(y,z) \wedge H(x,z)))]$

(There exists an x who is a relative of John, such that for any y , if y is a townsman, there exists a z who is a relative of y , and in addition x and z hate each other.)

Formula (21) is false in the model presented in Figure 5, while (18) is true and (17) is false. In other words, (21) follows from the strong reading of (1), but not from the weak reading. This argument indicates that in order to retain the natural inferential characteristics of (1) we should adopt the strong reading (17) rather than the weak reading (18).



Diag. 5. Does the picture represent the situation described in (20)?

However, this argument does not solve the problem, as it seems to be based on an arbitrary logical form assigned to (1). (21) states the existence of such a relative of John who hates a relative of each townsman, while — in my opinion — (20) does not determine any such thing. Sentence (20) merely says that each townsman has a relative who hates and is hated by a relative of John (i.e. each townsman may hate and be hated by a different relative of John). Therefore, (20) should rather be read as follows:

$$(22) \forall y[T(y) \rightarrow \exists x(R(\text{John}, x) \wedge \exists z(R(y, z) \wedge H(x, z)))]$$

(For any y , if y is a townsman, there exists an x who is a relative of John, and for some z , y is a relative of z , and x and z hate each other)

(22) is true in the model presented above and follows from the weak reading of the Hintikka sentence.

4. Barwise's test of negation normality

Jon Barwise proposes yet another test to determine whether a natural language sentence indeed has a non-linear logical form. His idea is based on the observation that for a non-linear quantifier it is impossible to construct a dual prefix by reorganising the relations within this prefix and dualizing the elementary quantifiers (cf. Krynicky and Mostowski 1995). We can prove even more: that for any branching quantifiers Q and Q' , if Q and Q' are dual,¹⁰ they are linear (Barwise 1979: 73).

¹⁰Quantifier Q is dual to Q' if for any formula ϕ , formulae $\neg Q\neg\phi$ and $Q'\phi$ are equivalent. For example, the prefix $\forall x\exists y\forall z$ is dual to the prefix $\exists x\forall y\exists z$.

Let us consider sentence (23) and its negation, which can be formulated in two ways — either by preceding the sentence with the "it is not the case that" operator, as in (24), or by changing the pronoun "every" to "some" and by reorganising the structure of the part of the sentence which is directly affected by these pronouns, as in (26). Let us assume after Barwise the practice of calling negations like (26) normal negations, and negations like (25), which refer to functions, not normal. If a sentence has a normal negation, we will call it a negation normal sentence, if not, then it is not negation normal. Naturally, in our example, sentence (23) is negation normal.

(23) Everyone owns a car.

(24) It is not the case that everyone owns a car.

(25) Not everyone has a car.

(26) Some people do not own a car.

Barwise suggests that there is an analogy between natural language and the language of elementary logic with branching quantifiers, which consists in the fact that natural language sentences having an essentially non-linear logical form cannot be negated normally, that is without referring to abstract objects: 'functions', 'assignments', 'choices', etc. According to Barwise, this makes it possible to formulate reasonable test criteria to check if a natural language sentence is truly non-linear (Barwise 1979: 56—57).

Barwise applies this test to sentence (1). He formulates two negations of (1), namely (28) and (29), which do not start with the words 'it is not the case that'. Sentence (28) does not refer to abstract objects and therefore Barwise considers it a normal negation, while sentence (29) is formulated by using the words "choose" and "assign", and thus — in Barwise's opinion — it is not negation normal. Then he asks proficient language users which of the sentences, (28) or rather (29), is equivalent to (27):

(27) It is not the case that some relative of each villager and some relative of each townsman hate each other.

(28) There is a villager and a townsman that have no relatives that hate each other.

(29) Any way of assigning relatives to each villager and to each townsman will result in some villager and some townsman being assigned relatives that do not hate each other.

If you prefer sentence (28), then it confirms Barwise's observations: "Again, in our experience, there is almost universal preference for" (28) (Barwise 1979: 58). Sentence (28) is equivalent to the negation of the weak reading of the Hintikka sentence. In other words, another 'empirical' argument proposed by Barwise is contradictory to Hintikka's suggestion concerning the logical form of sentence (1).

The division of natural language sentences with respect to their negation normality is, to put it mildly, rather imprecise. As opposed to formal languages, in natural language it is difficult to make an explicit reorganisation within the quantifier prefix, or to refer to the notion of function and other similar mathematical concepts. The complexity (difficulty) of negation is also not a good criterion (Barwise 1979: 60). For instance, formulating a logically correct negation of the following sentence:

(30) I solved all tests and managed to watch the film.

is a rather difficult task for many language users. A negation of such a sentence seems puzzling to them, although no reasonable person will argue that this sentence has a logical form impossible to express in elementary logic.

5. Sentences with the quantifier "most"

Barwise states: "The better a paper on branching quantification is, the more convincing is some example it contains." (Barwise 1979: 58). Above I have tried to show that the Hintikka sentence is not a convincing example to support the thesis that in order to make a logical analysis of natural language, we need a tool using branching quantifiers. At the same time, I have said that other works propose some better examples employing not only the quantifiers " \forall " and " \exists ", but also quantifying pronouns such as "most", "quite a few", "several", "many". The quantifier "most x such that $\varphi(x)$ fulfil $\psi(x)$ ", marked as $\text{MOST}x(\varphi(x),\psi(x))$, involves the least ambiguity. Elementary logic with an additional quantifier MOST is marked as L(MOST). The truth conditions for this quantifier are as follows:

$$M \models \text{MOST}x(\varphi,\psi)[\bar{u}] \text{ when } \text{card}((\varphi \wedge \psi)^{M,\bar{u},x}) > \text{card}((\varphi \wedge \neg\psi)^{M,\bar{u},x}),$$

$$\text{where: } \zeta^{M,\bar{u},x} = \{b \in |M| : M \models \zeta[\bar{u}(x/b)]\}$$

(Formula $\text{MOST}x(\varphi,\psi)$ is true in a model M at quantification \bar{u} if and only if the cardinality of the set of objects fulfilling the conjunction $(\varphi \wedge \psi)$ is greater than the cardinality of the set of objects fulfilling the conjunction $(\varphi \wedge \neg\psi)$).

The branching sentences with the quantifier "most" are for example:

(2) Most relatives of each villager and most relatives of each townsman hate each other.

(31) Most townsmen and most villagers hate each other.

(32) Most philosophers and most linguists agree with each other about branching quantification.

(33) Most footballers of FC Barcelona and most footballers of Manchester United exchanged shirts with each other.

(34) Most mobile phones and most chargers do not fit each other.

Examples (2) and (32) come from Barwise's paper (Barwise 1979: 60). (31) is a simplified version of (2), taken from an article by Mostowski (Mostowski 1994: 224). The remaining examples are my own and are meant to convince the readers that these discussion concerns actual situations in communication, and not only artificial linguistic contexts made up solely for the purpose of logic.

Let us remain within the relation village—town and let us take a closer look at sentence (31), which is of course equivalent to the sentence:

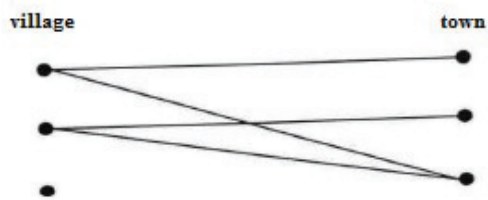
(35) Most villagers and most townsmen hate each other.

Let us consider two linear formulas which are possible candidates for the logical form of (31) and (35):

(36) $\text{MOST}x(\text{V}(x), \text{MOST}y(\text{T}(y), \text{H}(x, y)))$

(37) $\text{MOST}y(\text{T}(y), \text{MOST}x(\text{V}(x), \text{H}(x, y)))$

(36) is not equivalent to (37). In order to prove that, it is enough to construct a model (see Figure 6) in which (36) is true and (37) is false (Mostowski 1994: 225).



Diag 6. Model of the sentence (36) is not a model for a sentence (37).

In this case, we do not have at our disposal any equivalent of the weak reading, and therefore a natural candidate for the logical form of (30) is the following formula:

$$(38) \text{ MOST } xV(x)$$

$$[H(y,w)]$$

$$\text{MOST } yT(y)$$

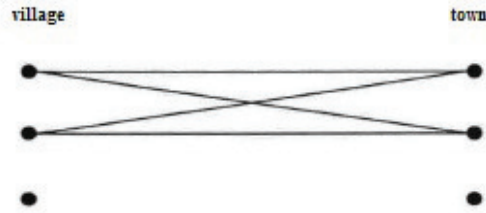
(38) is an essentially non-linear formula L(MOST) (cf. Mostowski 1994: 225).¹¹ The semantics of (38) will be expressed like this:

$$(39) \exists A \exists B (\text{MOST } x(V(x), A(x)) \wedge \text{MOST } y(T(y), B(y)) \wedge \forall x \forall y (A(x) \wedge B(y)) \rightarrow H(x, y))$$

(There exist predicates A and B , such that most x who are villagers are A , and most y who are townsmen are B , and in addition for any x and y , if $A(x)$ and $B(y)$, then x and y hate each other.)

This time even arguments *à la* Barwise speak for the non-linear reading. Can the readers construct in a natural way a negation of (31) without referring to 'assignments' or 'functions'? Will the readers consider Figure 6 or rather Figure 7 the adequate model for (31)? It is a shame that the methods proposed by Barwise have never been further specified and that, consequently, they cannot be applied to definite testing of the linearity of natural language sentences. Creating such precise tests would help us solve the dispute about the logical form of the Hintikka sentence.

¹¹Actually, in (Gierasimczuk and Szymanik 2009) another reading, the so called two-way quantification, has been proposed. This reading is weaker than the branching reading, and still seems to be empirically adequate.



Diag. 7. A model for the branching reading of (31)

Generally, I think that sentences like (31)—(34) are convincing examples to support the stronger thesis of Hintikka, i.e. that the logic of natural language is stronger than elementary logic.

6. Summary

The aim of this article was to point out that the arguments put forward in favour of (H') are not conclusive. On the other hand, one argument for the weak reading is certainly its simplicity. Furthermore, it might be possible to support it by some empirical experiments which would follow Barwise's guidelines.

Many interesting arguments have been used in the debate on the logical form of the Hintikka sentence, and they deserve attention on their own. First of all, Barwise has proposed methods of empirical testing of such problems — methods which, if properly systematised, may bring many interesting results. It seems particularly interesting to examine how people understand certain sentences by using schematic diagrams. Secondly, in the dispute on the reading of (1), we can clearly see the role of inferential relations in the research on the logical form of sentences and their links with the aspects of language studied by pragmatics. And finally, the participants of the debate have noticed the problem of computational complexity of semantic constructions of natural language.

Bibliography

1. Barwise, Kenneth J. (1979) "On Branching Quantifiers in English". *Journal of Philosophical Logic* 8, no. 1: 47—80.
2. Blass, Andreas and Yuri Gurevich (1986) "Henkin Quantifiers and Complete Problems". *Annals of Pure and Applied Logic* 32, no. 1: 1—16.
3. Gabbay, Dov M. and Julius M.E. Moravcsik (1974) "Branching Quantifiers, English and Montague Grammar". *Theoretical Linguistics* 1, no. 1: 140—157.
4. Gierasimczuk, Nina and Jakub Szymanik (2009) "Branching Quantification vs. Two-Way Quantification". *Journal of Semantics* 26, no. 4: 329—366.

5. Guenther, Franz and Jakob P. Hoepelman (1976) "A Note on the Representation of Branching Quantifiers". *Theoretical Linguistics* 3: 285—289.
6. Henkin, Leon (1961) "Some Remarks on Infinitely Long Formulae". In *Infinitistic Method*, 167—183. London: Pergamon Press.
7. Hintikka, Jaakko (1973) "Quantifiers vs. Quantification Theory". *Dialectica* 27, no. 3—4: 329—358 [also *Linguistic Inquiry* 5, no. 2 (1974): 153—177].
8. Hintikka, Jaakko (1976) "Partially Ordered Quantifiers vs. Partially Ordered Ideas". *Dialectica* 30, no. 1: 89—99.
9. Krynicki, Marcin and Marcin Mostowski (1995) "Henkin Quantifiers". In *Quantifiers: Logic, Models and Computation*, Marcin Krynicki, Marcin Mostowski, Lesław W. Szerba (eds.), vol. 1, 193—262. Boston (MA): Kluwer.
10. Mostowski, Marcin (1995) "Kwantyfikatory rozgałęzione a problem formy logicznej" ["Branching Quantifiers and the Problem of Logical Form"]. In *Nauka i język: prace ofiarowane Marianowi Przełęckiemu*, Mieczysław Omyła (ed.), 201—241. Warszawa: Zakład Semiotyki Logicznej UW.
11. Mostowski, Marcin and Dominika Wojtyniak (2004) "Computational Complexity of Semantics of Some Natural Language Constructions". *Annals of Pure and Applied Logic* 27, no. 1: 219—227.
12. Papadimitriou, Christos H. (1993) *Computational Complexity*. Reading (MA): Addison-Wesley.
13. Stenius, Erik (1976) "Comments on Jaakko Hintikka's Paper 'Quantifiers vs. Quantification Theory'". *Dialectica* 30, no. 1: 67—88.