Barbara Stanosz

PARADOX RESOLUTION

Originally published as "Rozwiązywanie paradoksów," Studia Semiotyczne 25 (2004), 27–31. Translated by Witold Hensel.

If we set aside cases where a statement is called paradoxical merely because it is surprising and clashes with common sense or previously accepted scientific theory (i.e., cases such as the opinion that Einstein's theory is paradoxical, common among our great grandfathers) then what remains on the battlefield are apparently valid inferences that lead from acceptable premises to unacceptable conclusions. The phrase *on the battlefield* seems apt because we tend to regard paradoxes as painful blows to human reason. We feel they must be parried or eliminated by means of iron-clad solutions. Often, different proposals to solve a single paradox enter into fierce competition.

Roughly speaking, paradox solutions fall into the following four categories: (A) Those that justify the thesis that the conclusion merely appears to be unacceptable when in fact it is quite natural and harmless; (B) Those that show that, appearances to the contrary notwithstanding, at least one of the steps in the inference is logically invalid; (C) Those that prove that at least one of the premises, which occurs explicitly or implicitly in the inference and seems to be acceptable, is actually false; the falsity of such a premise is sometimes argued for independently of the paradox, but sometimes the paradox itself is treated as a proof by contradiction of the premise's falsity; and (D) Those that show that what was taken to be an acceptable premise is semantically defective or, indeed, completely nonsensical (is not a statement), and thus cannot serve as a premise for any reasoning.

As an example of a type-(A) solution consider of how one may handle a version of the liar paradox attributed to Eubulides. Suppose, as this version invites us to, that the Cretan Epimenides says that all Cretans lie; from the assumption that he is telling the truth it follows that he is lying. Appealing to the principle of charity, we gloss over the difference in meaning between *to lie* and *not to tell the truth* and assume that what the sentence ”All Cretans lie” is meant to say is that no Cretan ever tells the truth. However, the conclusion of this inference merely says that Epimenides cannot truthfully assert that all Cretans are always lying. This conclusion should not be surprising, for if no Cretan ever tells the truth then no statement made by a Cretan can be true. The conclusion is harmless because from the assumption that Epimenides is not telling the truth it does not follow that he is telling the truth, ergo we do not get a contradiction.

Solution type (B) is difficult to apply because the authors of well-known paradoxes had usually taken great care to make their inferences logically valid. The only exception I know of is an analysis of Zeno’s paradox of the arrow. According to this paradox, the arrow cannot be in motion, since, at every given time, it is located at a particular place. On the analysis in question, Zeno’s reasoning involves a logically invalid inference from a statement of the form $\forall x \exists y \mathrm{R}(x, y)$ to a statement of the form $\exists y \forall x \mathrm{R}(x, y)$: i.e., from ”At every moment of its flight the arrow is located at a particular place” to ”There is a place at which the arrow is located at every moment of its flight.” However, this interpretation is quite unique in the literature devoted to the arrow paradox (see Ajdukiewicz 1965).

We employ type-(C) solutions to handle some other paradoxes by Zeno of Elea, which were allegedly intended to prove the impossibility of motion. One of those paradoxes, known as ”Achilles and the Tortoise,” leads to the conclusion that one runner (Achilles) will never overtake another runner who has had a head start (the tortoise) as long as the latter continues running, no matter how slowly. This conclusion is premised on the claim that, before Achilles has reached the point where the tortoise was a moment ago, the tortoise will have already moved to another place further down the track; this will happen over and over again infinitely many times. The fallacy rests on the implicitly assumed general claim the members of any infinite sequence of non-zero time intervals must add up to eternity; in reality, their sum can be finite, which solves the paradox.

We know that the premise under discussion is false from the mathematical theory of infinite series, independently of the paradoxical nature of Zeno’s conclusion. As an example of a type-(C) solution where the paradox itself is used to disprove one of its premises consider the way we analyze the following reasoning. Suppose that an aunt likes all those, and only those, members of her family who do not like themselves (or that a barber shaves all those, and only those, denizens of his town who do not shave themselves; both versions of this paradox are due to an unknown author or authors). Every possible answer to the question ”Does the aunt like herself?” implies the answer's own denial; consequently, the final conclusion is a statement of the form $p \wedge \sim p$. We solve the paradox by pointing out that it constitutes a proof by contradiction of the claim that no such aunt can exist in any family (and no such barber can exist in any town), for what the paradox shows is that the assumption of the existence of such an aunt or barber leads to contradiction.

The universally accepted solution to Russell's antinomy and the solutions to several other paradoxes in set theory are similar in character: all these paradoxes are now treated as proofs of the inexistence of certain sets. But the decision to treat them so was incomparably more dramatic than the decision to eliminate from our ontology the eccentric aunt or the monopolist barber. The realization that we need to abandon the assumption that every open sentence determines a set of objects that satisfy it had shaken the foundations of mathematics; it was also a painful reminder that our intuitions are not as trustworthy as we would like them to be.

Solutions of type (D) recommend themselves when we are confronted with paradoxes that clearly rely on the lexical or syntactic ambiguity of sentences or on the widespread vagueness of natural language expressions. Eubulides's paradox of the heap is a case in point. It is easy to agree that one grain of sand does not make a heap and that the difference of a single grain cannot determine whether or not something is a heap. But if you drop grains of sand one by one in the same place, sooner or later you will have made a heap. How is that possible if, as we have agreed, we cannot make a heap by adding a single grain to something that is not a heap? We reply by pointing to the vagueness of the word *heap*, a semantic defect of sorts (one that the word *heap* shares with many other words of our language) which makes it impossible for us to use the word in a consistent manner in some inferences — such as the one above.

We use the same kind of solution to tackle the modern version of the liar paradox, which is incomparably more troublesome than the original version discussed above. It is represented by the following reasoning. Let the letter S stand for the statement:

Statement S is false.

Now ask: Is S is true? What we get is a contradiction, for every answer implies its own denial (given the dichotomy of truth and falsehood): if S is true then S is false, and if it's false then it's true.

It is not easy to identify a defect in the notions of truth and falsehood (as well as other semantic notions, including that of reference, which yields a similar paradox) that we could blame for the contradiction. And it is not easy to simply stop using these notions as nonsensical — as some authors would have us do — or to restrict their use to selected contexts, of which we would be confident that they did not engender a contradiction. Both these solutions seem too radical. And what I have in mind are not the practical problems with enforcing such restrictions in philosophy or everyday communication if one is confronted with people who do not feel particularly inclined to follow the rules of logic; rather, the trouble is that accepting such strictures might damage some discourses the logician is inclined to treat as cognitively valuable, especially now that we know, thanks to Tarski, that truth is definable for many of them.

The common feature of most (variously formulated) solutions to the liar paradox is that they treat semantic notions as systematically syntactically ambiguous. What we actually have, instead of two notions ”true” and ”false,” are infinite families of notions: ”true_0,” ”true_1,” ”true_2,”..., ”false_0,” ”false_1,” ”false_2,”..., and, furthermore, when you have a sentence predicating truth or falsity about a sentence that itself features ”true” or ”false” with the subscript x, syntactic coherence demands that it contain the appropriate term with the superscript $x + 1$. In light of this requirement, what we have marked as S above is not a well-formed sentence of any language. The right answer to the question ”Is an utterance that attributes falsity_0 to itself true_1 or false_1?” is ”No, such an utterance makes no sense.” This answer has no paradoxical consequences (the answer that the utterance is false_1 would not be paradoxical either, though it would be false_2).

Of course, this kind of solution is not a description of the actual use of semantic concepts in any of the previously existing languages; rather, it is a prescription of how to use semantic concepts in order to avoid contradiction — it is a piece of advice addressed to all those who construct languages with semantic concepts. However, there is no suggestion here that we should modify our use of semantic notions in natural language to bring it in line with this idea; logicians who offer this kind of solution see the liar paradox as a price natural language has to pay for the indispensable universality of communication functions it fulfils. But some philosophers and linguists try to defend the ordinary notion of truth against the charge of inconsistency.

They usually seek to prove that S is either ungrammatical or does not constitute a complete, autonomous unit of natural language and, as such, cannot be true or false; in a sense then the meaningfulness of S is being questioned here along with the role *S* plays in the liar paradox. How can one secure such a claim? The task seems hopelessly difficult. Above all, if it is to escape the charge of being *ad hoc*, such a defense of the ordinary notion of truth must cast doubt on the meaningfulness of *S* along with a whole class of expressions with a similar structure. Yet, even if we concede that the status of *S* as a sentence of natural language is dubious, there are a multitude of expressions that bear close structural resemblances to *S* but which are often used as independent statements and raise no suspicions. More specifically, one should not dismiss as senseless all self-referring statements, because one would thereby reject many perfectly natural sentences (which do not generate contradictions), such as the question ”Can you hear what I'm saying?” uttered while testing the microphone.

This appeals to a particular type of grammatical description (namely, generative grammars) that exclude expressions such as *S* from the set of well-formed sentences and do not constitute a plausible argument: any natural language is describable in terms of many different grammars, which, though better or worse from the pragmatic point of view, are theoretically on a par even if they do not generate the exact same set of sentences. For any such description is an idealization; it arbitrarily sharpens the notion of a sentence of a given language. Not even the best empirical evidence will yield a determinate answer in this matter. However, a grammar is not an adequate description of language if it excludes from the set of sentences (as nonsensical or non-autonomous) many expressions used in communication as independent sentences.

Bibliography

1. Ajdukiewicz, Kazimierz (1965) *Zmiana i sprzeczność*. In *Język i poznanie*, vol. 2, 90-106. Warszawa: PWN.