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Two Models of Propositional Structure

Abstract This paper is a comparison of two structural theories of propositions: the theory proposed by Kazimierz Ajdukiewicz in the 1960s and the theory developed by Jeffrey King at the beginning of the 21st century. The first section of the paper is an overview of these theories. The second part is a detailed discussion of significant similarities shared by them. In this section, I also identify and analyze ways in which these theories differ and attempt to determine if these differences are substantial or apparent. The last part is an attempt to determine if the discussed theories are capable of coping with the Benacerraf Problem.

Keywords Kazimierz Ajdukiewicz, Jeffrey King, proposition, propositional structure, structural theories of propositions, the Benacerraf Problem, truth conditions

Realists regarding logical propositions (cf. Loux 2003) disagree as to the set of properties attributable to propositions.² One of the most contentious issues in the debate regarding the nature of propositions is their structure. Generally speaking, the locus of controversy is the question of the divisibility of propositions into constituents. Philosophers holding that propositions are divisible are referred to as the proponents of the structural theory of propositions. According to this theory, propositions are structured objects, and the structure of a proposition corresponds, to a lesser or greater degree, to the structure of the sentence expressing it. The alternative approach to the structural theory is the functional theory, according to which propositions are functions relating possible worlds to truth values.

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²Perhaps the only property that does not arouse controversy is abstractness. See Kirkham (2001, p. 57), Loux (2003, p. 137), or Lycan (2002, p. 80).
The functional theory is traditionally taken to derive from Rudolf Carnap’s conception of extension and intension (Carnap 1947/2007). In this conception, propositions are identified with intensions of sentences, and truth values with their extensions. It is required that the intension of a sentence unequivocally determine its extension on the one hand, and on the other, that it be possible that two sentences sharing the same extension possess different intensions. Over time, an entire tradition emerged of identifying intension with the function \( I : W \rightarrow \{0, 1\} \), that is, one relating the set of possible worlds (\( W \)) to truth or falsity.\(^3\)

The main group among the proponents of the structural theory of propositions are philosophers developing Bertrand Russell’s position presented in *The Principles of Mathematics* (Russell 1903/2008). According to this position, a proposition is, roughly speaking, a complex of objects, properties and relations spoken of in the sentence expressing this proposition insofar as these objects possess these properties and remain in these relations. Russell’s position is sometimes referred to as direct realism because, according to it, a proposition expressed by the sentence \( xyz \) is in principle\(^5\) constituted by objects \( x, y \) and \( z \), not by their representations, concepts, or the meanings of expressions designating them. In other words, one constituent of the proposition expressed by the sentence *Russell is British* is Russell himself – the flesh and blood human being. Propositions of this kind are referred to as singular propositions, and expressions introducing their designates (not their meanings) into these propositions, as directly referential.

A theory inspired by this position and at the same time markedly different from other Russellian conceptions\(^6\), has recently been proposed by Jeffrey King (2007)\(^7\). One of the distinctive features of this conception is

\(^3\)The most important proponents of the functional theory have been Lewis and Montague. A significant extension of the functional theory is so-called two-dimensional semantics developed by Stalnaker, Kaplan, and Chalmers. A solid Polish language discussion of issues related to the functional theory can be found in (Ciecierski 2003), and those related to two-dimensional semantics, in Odrowąż-Sypniewska (2006, 330–336).

\(^4\)The expression “insofar as” should be understood in a sense independent of the cognitive act performed by the subject since Russell considered propositions to be objective entities, independent of the mind. Compare Russell (1902/2008, p. 33) and Makin (2000, p. 11).

\(^5\)The infamous denoting concepts being the exception. Compare Russell (1903/2008, p. 5).

\(^6\)Soames, Salmon, Richard and sometimes Kripke and Kaplan are considered to be the continuators of those other conceptions. See Deutsch (2008).

\(^7\)King later proposed a revised version of his conception (King 2014). It differs from the 2007 formulation in that much more attention is given to the impact of context on the proposition expressed by a sentence. However, since the fundamental ideas have
its refusal, *contra* many theoreticians of singular propositions, to identify propositions with any kind of formal constructs. King brings propositions back to earth, so to speak, by identifying them instead with a special kind of facts. In this paper, I compare King’s famous conception to a less discussed, including in Poland, theory of propositions as functions proposed by Kazimierz Ajdukiewicz (this theory is not a functional theory in the sense indicated above). I am of the opinion that they have enough in common for their juxtaposition to be interesting not only for the historian of philosophy but also for the contemporary philosopher of language. The reason is that their comparison can help shed light on certain nontrivial issues related to the problem of propositional structure and render salient some of the consequences of choosing a particular model thereof.

The first part of the paper is a detailed discussion of the two conceptions. The second part is dedicated to their comparison; here, I indicate similarities and apparent, as I am going to argue, differences between them. In the final part of the paper, I test both conceptions in light of the Benacerraf Problem. I argue that neither of them passes the test since the proponents of both conceptions are forced to introduce *ad hoc* solutions, or to accept difficult consequences, in order to tackle this problem.\(^8\)

1. Conceptions from King and Ajdukiewicz

1.1. King’s conception of propositions as facts

According to King’s conception, the proposition expressed by the sentence *Rebecca swims* is a fact, although not the fact that might come to mind as corresponding to this sentence at first glance, that is, not the fact that Rebecca has the property of swimming. The fact of Rebecca’s having the property of swimming is a truth-maker of the fact-proposition expressed by this sentence\(^9\), but the two facts are not the same. Significantly, as King points out, if Rebecca did not in fact swim, the fact of her having the property of swimming would not obtain (would not exist); the proposition

\(^8\)I thank Tadeusz Ciecierski for first pointing out that Ajdukiewicz’s and King’s conceptions share similarities. He mentions this in passing in Ciecierski (2012).

\(^9\)In other words, the proposition-fact [that Rebecca swims] is true if and only if Rebecca instantiates the property of swimming.
under consideration, on the other hand, would exist, although in these circumstances it would of course be false (cf. King 2007, p. 26).

The methodological background of King’s theory is a syntactic sentence analysis based on Chomsky’s categorial grammar, and more precisely, on a version of the latter called the minimalist program. In particular, King uses the method of representing the real (deep) syntax of sentences using so-called trees—a method well entrenched in the tradition of syntactic investigations. For example, the syntax of a simple subject-predicate sentence such as *Rebecca swims* can be represented in the following way:

Tree 1. (the sentential relation)

King refers to the relation responsible for binding simple expressions into a complex whole, that is, a sentence, here graphically represented by the branches of the tree, as the sentential relation (King 2007, p. 29). There are two options concerning the nature of sentential relations, according to King. We can assume either that the sentential relation is a nondefinable primitive concept or that its nature is currently impossible to explain, although it might be explained in the future by means of cognitive and neurological concepts (King 2007, p. 47–50). King admits that he is inclined toward the latter option, but the first one does not diminish the value of the proposed description of propositional structure, in his opinion.

According to King, objects constituting fact-propositions include, first of all, properties and relations (such as the property of swimming), and secondly, individual objects, including macroscopic physical objects (such as Rebecca or Mount Everest). A complete tree representing the fact-proposition expressed by the sentence *Rebecca swims* looks like this:

The two branches converging at the root of the tree (its highest point) and shaping the entire structure represent the sentential relation (in short, R) binding the relevant simple expressions into a sentence, as featured in Tree 1. “Rebecca*” represents the physical individual, that is, Rebecca, and “swims*”, the property of swimming conceived as an abstract object.10 The

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10The asterisks serve to emphasize the fact that it is Rebecca as such and swimming as such that make up the syntax of the proposition, not meanings or concepts etc.
Two Models of Propositional Structure

I

Tree 2

lines linking “Rebecca*” and “swims*” to the small circles represent the semantic relations obtaining between the expression “Rebecca” and Rebecca (or Rebecca*) and the expression “swims” and the property of swimming (or swimming*). These semantic relations are relations of reference, or designation, that is, relations obtaining between expressions and objects constituting their reference.

The aforementioned circles represent a relation referred to by King as joint instantiation. It holds between two properties of the expressions making up the analyzed sentence: the property of referring to a designate forming a part of the fact-proposition and the property of constituting a particular node in the relation R characteristic of the discussed sentence. For example, the circle located on the left branch of the tree represents the fact that two properties of the expression “Rebecca” are jointly instantiated: first, that the expression refers to Rebecca, and second, that the expression constitutes the left node in the relation R, its right node being the expression referring to the property of swimming. Graphically speaking, joint instantiation is the point at which the syntactic relation meets the semantic relations to jointly make up the proposition (this can be seen from the suggestive location of the symbols for joint instantiation in the structure of the tree).

The rectangles located on the branches of the tree representing the relation of reference represent the relativization of the arguments of this relation to the context in which the sentence was used. In the example under consideration, this is meant to signal that “Rebecca” designates Rebecca in this context and “to swim” designates swimming in this context. King refers to the complex relation obtaining between the terms of the
Two Models of Propositional Structure

A proposition – the relation constituted by the syntactic relation and the (context relative) semantic relations described above – as the propositional relation (in short, P).

The last part of the discussed model is the broken line and the relation I located at its terminal node. The letter “I” represents an instruction, as King calls it, and the broken line linking R and I, the fact that I is encoded by R (one could say that the relation R is the carrier of the instruction I) ((King 2007, pp. 34–38)). An instruction indicates the most general rules for the determination of the truth conditions of the proposition it is an element of. In the case of the proposition discussed here and expressed by Rebecca swims₁¹, the instruction reveals that the proposition can be deemed true if and only if Rebecca has, or instantiates, the property of swimming. Put more generally, the instruction determines the configuration in which the objects and the properties constituting the reference of the expressions located on the individual branches of the relation R must remain in order for the analyzed proposition to be true. In the case of propositions expressed by simple subject-predicate sentences, the instruction usually determines the proposition to be true if and only if the object constituting the reference of the expression located on one branch of R instantiates the property constituting the designate of the expression located on its other branch.₁²

The idea of introducing the instruction I into the structure of the proposition should become clearer once we imagine a natural language in which the structure of the proposition expressed by the sentence Rebecca swims belonging to this language looks exactly the same as the structure illustrated in Tree 2 except for one difference: instead of I, the structure features ˜I, determining that the proposition is true if and only if Rebecca does not have the property of swimming. It is obvious that this hypothetical language is very different from Polish (and, most likely, the great majority of natural languages). However, its examination points to an important idea in King’s conception – namely, that the relation P constituting the structure of the

₁¹ In fact, the instruction I can be seen as an element of the proposition responsible for determining which constituent of its truth conditions relates to the subject of the proposition and which to the predicate.

₁² In King’s approach, propositions can also be represented in an abbreviated linear form (omitting joint instantiation, which must be taken to feature implicitly), where the shape of the relation R is represented by a sequence of square brackets, and C (relativization to context) and I (instruction) are represented by letters located at the beginning of the notation. For example, the proposition [that Rebecca swims] can be represented as {C, I, [Rebecca* [swimming*]]}, and the proposition [that Mont Blanc is shorter than Mount Everest] as {C, I, [Mont Blanc* [being shorter than* [Mount Everest*]]]}.

Studia Semiotyczne — English Supplement, vol. XXIX 87
proposition is insufficient, in and of itself, to determine even the most general truth conditions of this proposition; the proposition, as a carrier of truth, must also feature an element providing hints regarding its truth conditions. According to King, the syntactic relation R encoding the instruction I is such an element. At first glance, it might seem controversial to link truth conditions – even most generally construed, as is the case in I – to the level of syntax. The controversy subsides, in my opinion, once the role of syntax is taken to be such that the syntactic characteristics of expressions somehow determine their semantic characteristics; more specifically, the syntax determines the order of the semantic values of the particular expressions making up the sentence (that is, their designates). Knowledge of the syntax allows one to recognize this order – the order is in a sense encoded in the syntax.

Having explained what is represented by the particular elements of the fact-proposition model, King gives the following definition of a proposition:

The proposition expressed by a sentence of the form “xyz...” is the following fact: there exist a context C and expressions x, y, z of a language L whose semantic values in X are objects X*, Y*, Z*... and these expressions occur in a particular order determined by the sentential relation R encoding the Instruction I (King 2007, pp. 39, 42).

This formulation is surprising since a proposition is herein identified with the existence of expressions bearing certain syntactic and semantic characteristics, whereas the fact-proposition model presented earlier did not feature expressions at all. Moreover, in another part of his book King gives another characterization of a proposition, corresponding to what is presented using trees:

[...] the facts that are propositions came into existence in part as a result of lexical items acquiring semantic values and syntactic relations coming to encode certain functions (King 2007, p. 65).

The two explications of the notion of a proposition given by King are not mutually exclusive but they certainly differ, the difference being more than verbal. According to the first explication, a proposition is the existence of expressions etc.; according to the second one, it is the fact that the designates of the relevant expressions remain in a certain order. In the remainder of this

13I thank an anonymous reviewer of this article for drawing my attention to this difficulty in King’s conception.
paper, I refer to the latter interpretation, that is, I assume that, according to King’s theory, the proposition expressed by a sentence of the form “$\psi$ is $\varphi$” is the fact that the designate of $\psi$ and the designate of $\varphi$ remain in a particular order because they are the designates of these expressions. This decision is occasioned, in the first place, by the fact that the second interpretation predominates in King’s book, while the first only occurs in the initial parts of the text—it might thus be read as a not too fortunate initial statement. Secondly, there is no doubt that King presents his conception as a structural theory, and identifying a proposition with the existence of expressions bearing certain characteristics does not befit this strategy. King simply does not analyze the structure of the existence of such expressions.

Having established that, we can conclude that, according to King’s standpoint, a fact-proposition is something else than the fact intuitively assumed to be the proposition’s truth maker. The fact that Rebecca and swimming stand in the relation P – comprising the relation of designation, the sentential relation, and the relation of joint instantiation – is certainly different from the fact of Rebecca’s having the property of swimming. Both facts feature Rebecca and the property of swimming, but these stand in different relations in the first and the second fact.

The proposition [that Rebecca swims] identified with an appropriate fact is thus true if and only if Rebecca has the property of swimming or if Rebecca belongs to the extension of this property. In light of this, general truth conditions, as construed in King’s conception, can be characterized in the following way (assuming standard I):

\[
\text{The proposition expressed by a sentence of the form } \varphi(a_1, a_2, \ldots, a_n) \\
\text{is true if and only if the objects } a_1, a_2, \ldots, a_n \text{ belong to the extension of } \varphi.
\]

### 1.2. Ajdukiewicz’s structural-functional conception of propositions

A conception similar in its general outline to the one developed by King had been presented several decades earlier by Kazimierz Ajdukiewicz (Ajdukiewicz 1967/1971). Ajdukiewicz’s approach is based on his own syntactic analysis of sentences. The method in question consists in assigning to each expression in the syntax of a sentence an unequivocal description of its syntactic position in this sentence (compare Ajdukiewicz 1960/1985).

According to Ajdukiewicz, the proposition expressed by a sentence can be characterized as a function relating each syntactic position to exactly one
object constituting the reference of the expression occupying this position in this sentence. For example, the proposition expressed by the sentence:

\[
\text{Mont Blanc is shorter than Mount Everest}
\]

\[
(1,1) \quad (1,0) \quad (1,2)
\]

is a function relating position (1,1) to Mont Blanc, position (1,0) to the relation of being shorter than, and position (1,2) to Mount Everest. Since every function is identical with an appropriate set of ordered pairs, the function constituting the proposition expressed by the above sentence is a set of the following form (cf. Ajdukiewicz 1967/1971, pp. 122–123):

\[
\{\langle (1, 1), \text{M. Blanc}^* \rangle, \langle (1, 0), \text{being shorter than}^* \rangle, \langle (1, 2), \text{M. Everest}^* \rangle\}
\]

The following formulation can thus be used to explicate Ajdukiewicz’s concept of proposition:

The proposition expressed by a correctly constructed sentence \( S \) is a function \( \alpha : X \to Y \), where \( X \) is the set of the syntactic positions of the expressions constituting \( S \), and \( Y \) is identical with the universum.

Ajdukiewicz’s theory assumes an isomorphism between the structure of a true proposition expressed by a sentence and the ordering of the fact described in this sentence:

The assignment of syntactic positions to objects may or may not agree with the respective positions of these objects in reality. If the sentence stating a given proposition is true, then the respective positions of the objects spoken of in this sentence in reality agree with the syntactic positions assigned to these objects in the proposition stated by the sentence. In such a case, it seems natural to call the proposition stated by the true sentence a fact (Ajdukiewicz 1967/1971, p. 124).

In short, according to Ajdukiewicz, if a sentence expresses a true proposition, the order of the expressions in this sentence corresponds to the order of their designates in the world. The proposition expressed by a sentence, in turn, is a relation assigning designates to the syntactic positions of the expressions constituting this sentence. Ajdukiewicz’s syntactic analysis is based on the distinction of expressions playing the role of operators and those
playing the role of arguments. Roughly speaking, every situation comprises the “protagonists” of this situation, their properties, and the relations that bind them. The distinction into operators and arguments at the level of the sentence corresponds to the shape of the situation: the designates of the argument-expressions correspond to the “protagonists” of the situation, and the properties and relations comprising the situation constitute the designates of the operator-expressions. Given this, the general truth conditions of a given proposition can be characterized as follows:

The proposition $\alpha$ expressed by a sentence $S$ is true if and only if, for each compound expression $E$ distinguishable in $S$, it is the case that the objects constituting the designates of the expressions occupying argument positions in $E$ stand in the relation\textsuperscript{14} constituting the designate of the expression occupying an operator position in $E$, and these objects stand in this relation in an order corresponding to the order determined by the numbering of the syntactic positions of the argument-expressions.

For example, the proposition expressed by the sentence "Rebecca swims" is true if and only if the designate of the expression occupying the position of the argument has the property constituting the designate of the expression occupying the position of the operator – that is, if Rebecca has the property of swimming.

Thus formulated truth conditions faithfully reflect Ajdukiewicz’s approach. It is also not difficult to see that they constitute a particular version of a more general formulation according to which the proposition expressed by a sentence of the form $\varphi(a_1, a_2, \ldots, a_n)$ is true if and only if the objects $a_1, a_2, \ldots, a_n$ belong to the extension of $\varphi$ – the same as the formulation entailed by King’s theory.

Ajdukiewicz’s theory of propositions is a rare case in that it combines characteristics of both the structural and the functional theory of propositions. On the one hand, a proposition is determined largely by the structure of an appropriate sentence – this structure determines what is bound by the proposition conceived as a special kind of relation. In other words, the set identified with a proposition contains elements corresponding to the particular constituents of the sentence expressing this proposition. This aspect of Ajdukiewicz’s theory clearly brings it closer to the structural approach. On the other hand, one ought not to forget that Ajdukiewicz identifies a

\textsuperscript{14}For brevity, I assume that properties are one-argument relations.
Two Models of Propositional Structure

proposition with a certain kind of function, and this is characteristic of functional theories.

If Ajdukiewicz’s theory is functional, it is certainly nonstandard. According to him, a proposition is a function relating arguments in the form of the syntactic positions of expressions constituting the sentence expressing this proposition to values in the form of the designates of these expressions. In standard functional approaches, on the other hand, a proposition is a function determining the truth value of the sentence expressing this proposition for each possible world, that is, a function from possible worlds to the two-element set containing truth and falsity. To put it another way, according to Ajdukiewicz – and structural approaches in general – a proposition is constituted by what it is about, and the truth value is predicated of the proposition. In typical functional theories, in contrast, the truth value is, in a sense, a constituent of the proposition.\(^{15}\) Given this, it seems that Ajdukiewicz’s conception is closer to structural standpoints than it is to functional ones. Moreover, as I intend to argue, it has much in common with King’s conception.

2. Comparison

2.1. Similarities

Although the respective theories by Ajdukiewicz and King were presented at different times and against fundamentally different philosophical backdrops, I am of the opinion that they are based on the same overall idea. There are two nontrivial differences between them, but they have enough in common for their comparison to be worthwhile. I think that this comparison can help bring to light certain specific issues concerning logical propositions in general and singular propositions in particular.

Let us consider the sentence *Rebecca swims*. In King’s conception, the proposition expressed by this sentence is a fact consisting in the obtaining of the relation \(P^{16}\) which binds two objects (Rebecca and the property of swimming) by means of the relations that constitute it. One could say that King begins constructing his tree by determining the relation \(R\) which binds

\(^{15}\)Of course, insofar as we permit that proposition-functions be considered as ordered pairs whose elements are possible worlds and truth values.

\(^{16}\)For simplicity’s sake I temporarily ignore the instruction \(I\) featuring in King’s model. The instruction is a part of the proposition in this model but not a part of the relation \(P\). Since \(I\) is encoded by \(R\), which is a part of \(P\), this simplification seems to be acceptable.
the appropriate linguistic expressions and to which the relevant semantic relations are added during subsequent analysis so that the relation P can emerge. Let us see how an analogous procedure of a model for a proposition (and thus, of determining the structure of this proposition) looks like in the case of Ajdukiewicz’s conception.

Ajdukiewicz’s syntactic analysis of the sentence *Rebecca swims* looks like this:

\[
\begin{array}{c}
\text{Rebecca} \ \swims \\
(1, 1) \ \ (1, 0)
\end{array}
\]

As a result of this step, the syntactic relations obtaining between the expressions constituting this sentence – captured using the relation R in King’s model – have been determined. We can thus move onto the next part of King’s tree, namely, the part where the appropriate semantic relations are represented (i.e. the relations of reference obtaining between the name “Rebecca” and Rebecca and the expression “swims” and swimming).

According to King’s approach, the expression entering the given semantic relation is identified by reference to its syntactic characteristics, that is, by determining its location on one of the branches of the tree. For example, it is indicated that Rebecca is the designate of the expression located on the left branch of Tree 1, that is, of the name “Rebecca”. An analogous step can be found in Ajdukiewicz’s analysis. Here, the proposition is taken to be the function relating each syntactic position distinguished in the sentence expressing it to the object constituting the designate of the expression this syntactic position. To determine the syntactic position of an expression is thus, no more no less, to identify this expression by reference to its syntactic characteristics (it is impossible for two expressions to occupy the same syntactic position). For instance, Rebecca is assigned to position (1,1) in our example because this position is occupied by the name “Rebecca” referring to Rebecca.

In King’s conception, the compounding of the syntactic relation and the semantic relations – that is, the compounding of the relation R and the relation of reference – gives in effect the relation P which can be thought of as the structure of the proposition. In Ajdukiewicz’s conception, the compounding of the analogous relations – that is, the assignment of syntactic positions to the expressions featured in the sentence and the subsequent assignment of appropriate designates to these positions – determines a
function identical with the set $A$:

$$A = \{(1, 1), \text{Rebecca}^*\}, \{(1, 0), \text{swimming}^*\}.$$\(^{17}\)

The set $A$ thus plays the same role in Ajdukiewicz’s conception as does Tree 2 in King’s theory. Namely, both represent the relation obtaining between the syntactic positions distinguishable in a sentence and the designates of the expressions occupying these positions. This ordering of the designated objects by reference to the syntactic characteristics of the expressions designating them is at the core of a proposition, according to both conceptions.\(^{18,19}\)

2.2 (Apparent) differences

Regarding the constitution of a proposition, the two conceptions differ in a twofold manner. The first difference is that Ajdukiewicz’s set $A$ is slightly poorer in information than Tree 2, its analogue in King’s conception. Ajdukiewicz’s model does not account for three elements considered by King: context, instruction, and the relation of joint instantiation.

As far as the relation of joint instantiation is concerned, it seems legitimate to claim that it is inscribed into $A$. The set is determined in such a way that it is clear that the term “Rebecca” refers to Rebecca and that it constitutes the first argument of the operator in the form of the expression referring to the property of swimming—precisely these two properties of the term “Rebecca” are captured in King’s model as the relation of joint instantiation.\(^{20}\)

\(^{17}\)It might be worth noting that both in King’s model and in Ajdukiewicz’s conception the last stage of the analysis of the nature of the proposition (Tree 2 in King and Set A in Ajdukiewicz) does not feature expressions themselves. The transition from Tree 1/syntactic analysis to Tree 2/Set A consists, among other things, in the removal from the model of the names of the expressions making up the analyzed sentence and limiting the model to the syntactic characterization of the expressions on the one hand, and to their designates, on the other.

\(^{18}\)This clearly differentiates the two positions from many other versions of the structural approach to propositions (e.g. those of Soams or Salmon) which assume that the structure of the proposition is somehow correlated with the structure of the sentence but do not incorporate this assumption into their actual models of propositions.

\(^{19}\)It might be worth noting that the postulate to reflect the structure of the sentence in the structure of the proposition is dictated mainly by the desire to avoid the problem of an imprecise identification of propositions faced by functional theories.

\(^{20}\)As mentioned earlier, joint instantiation is responsible for the compounding of the syntactic relation and the appropriate semantic relations. What is in a sense analogous to this in Ajdukiewicz’s model is the apprehension of a given syntactic position and an appropriate designate as an ordered pair.
The question of context and instruction is more difficult. The instruction contains information concerning the general truth conditions of a proposition and is encoded by the syntactic relation R constituting a part of the propositional relation. Bluntly speaking, the relation R determines the way in which the designates of the expressions bound by R must be connected in order for the analyzed proposition to be true. In the case of the sentence *Rebecca swims*, its syntax determines the fact that the proposition expressed by this sentence is true if Rebecca has the property of swimming. Ajdukiewicz’s method has certain advantages over King’s model because here the relationship determining the truth conditions of a proposition is contained in the very syntactic analysis of the sentence expressing it, based on the distinction into arguments and operators. A proposition is true if the designate of the expression constituting the argument, or the designates of the expressions-arguments, satisfies the condition expressed by the term or phrase functioning as the operator. In order for the proposition about Rebecca to be true, Rebecca – as the designate of the expression-argument – must instantiate the property expressed by the operator “swim”. This kind of relationship between the designates of the expressions making up a sentence is thus taken into account already at the level of syntactic analysis. Therefore, in Ajdukiewicz’s approach there is no need to “glue” an extra instruction onto the propositional relation (this step is required in King’s approach). On the other hand, each analysis carried out using Ajdukiewicz’s method encodes the same kind of instruction – a proposition is true if the designates of the expressions-arguments satisfy or fall under what is expressed by the operator. It is thus impossible to encode an instruction imposing that the proposition expressed by the sentence *Rebecca swims* is true if and only if Rebecca does not instantiate the property of swimming. Owing to the fact that King treats the instruction I as encoded by the relation R, but also external and autonomous relative to it, he can successfully represent different instructions governing the truth conditions of a proposition.\(^\text{21}\)

\(^{21}\)It is legitimate to ask at this point if a theory of propositions must in fact account for the possibility to encode different instructions in a proposition. There are two sub issues here. On the one hand, there is the empirical question of whether there exists a (natural) language in which the proposition [that Rebecca swims] is true if Rebecca does not have the property of swimming. On the other hand, one can doubt, on theoretical grounds, if such a language is at all possible. Its users would certainly possess a different notion of truth from ours. One could say that, for them, the predicate “true” is a synonym of our predicate “false”. This is not the right place to offer a detailed discussion of the concept of truth but, in light of the above, it is justified to claim, in my opinion, that King’s concept of instruction is at best vague.
As far as accounting for dependence on context (sensitivity of the reference of given expressions to context) in a proposition is concerned, there is, in short, no room for it in Ajdukiewicz’s model. However, there is no reason why appropriate contextual parameters could not be introduced into the description of the proposition – in such a case, one would assume that a given sentence expresses this or that proposition in this or that context. There is also no doubt that context must play a role in determining the designates entering into ordered pairs involving expressions sensitive to context, especially indexical expressions. There is a fundamental difference between this kind of approach and King’s conception: King takes relativization to context to be a constituent of the proposition, not a part of its description. King (2007, p. 39) is convinced that not accounting for context in a proposition must yield a theory that does not permit propositions expressed by sentences containing expressions sensitive to context. I must leave the highly complex question of whether context is best seen as external to the proposition (as in Ajdukiewicz) or as an integral constituent thereof (as in King) open. Regardless of the solution, this constitutes a clear difference between the two theories.

Another difference regards, to put it in general terms, the ontological status of propositions. King is very clear that in his theory’s propositions are not identified with any kind of formal constructs, and thus, in particular, that they are not identified with functions. A proposition, according to King, is a special kind of fact: the fact that a certain relation obtains between certain objects. Importantly, the proposition is not identical with this relation. For example, the propositional relation P illustrated in Tree 2, linking Rebecca and the property of swimming, is not identical with the fact-proposition [that Rebecca swims]. As has been shown, the (rough) equivalent of the propositional relation in Ajdukiewicz’s theory is a function identical with the set A whose elements are ordered pairs containing specific kinds of objects. However, Ajdukiewicz does not claim that a proposition should be identified with the fact of the occurrence of this function or the fact of the existence of the set A; in his conception, the proposition expressed by the sentence Rebecca swims is that function, and thus, is the set A. In short, according to Ajdukiewicz, a proposition is a relation, and according to King, it is the fact that a certain relation obtains. That said, it is worth considering if this difference is in fact as fundamental as it might seem at first glance rather than being purely verbal.

Ajdukiewicz was certainly not a flippant philosopher. If he said that a proposition-function can be identified with a set of appropriate ordered pairs,
then it should be concluded that that is what he meant, not merely that the function can be so represented. If we stick to the letter of Ajdukiewicz’s argumentation, then, the difference between his theory and King’s might indeed be fundamental.

That said, I am of the opinion that another reading of Ajdukiewicz’s conception – one close enough to his intention – is possible. It is based on an alternative interpretation of the concept of function. Ajdukiewicz assumed a commonly accepted and frequently employed set-theoretical interpretation of this concept according to which a relation, in particular a function, is a set of ordered pairs of elements constituting the arguments of this relation. However, if we treat a function in a less “logical” and more “ontological” manner, we can characterize it as a *sui generis* mechanism, process, and even fact – the fact that an assignment occurred where objects belonging to a certain set are related to objects belonging to another set. In light of this, Ajdukiewicz’s proposition can thus be identified with the occurrence of an assignment of certain objects to appropriate syntactic positions.22 If we assume that the interpretation of Ajdukiewicz’s conception according to which every proposition is a proposition about (among other things) certain syntactic positions is not correct (see below), and if we accept the aforecited alternative understanding of the concept of function, we get an approach according to which a proposition is identical with the fact of the occurrence of a certain relation between objects constituting the designates of the expressions making up the sentence expressing this proposition. This relation, on the other hand, is the compound of the syntactic relation (captured via syntactic positions) binding expressions making up the given sentence and the semantic relations obtaining between the particular expressions making up this sentence and their designates. It is not difficult to see that this summary of Ajdukiewicz’s standpoint overlaps with the characterization of propositions offered by King. It is doubtful that King is familiar with Ajdukiewicz’s conception, but if it were the case, one could convincingly argue that his theory is a development of Ajdukiewicz’s conception, as interpreted here.23

22 This interpretation is supported by the expression used by Ajdukiewicz to characterize a proposition. Namely, he writes that a proposition is a function establishing the assignment of syntactic positions to designates (cf. Ajdukiewicz 1967/1971, pp. 123, 124).

23 One should remember that the difference concerning the constituents of a proposition indicated by King but missing from Ajdukiewicz’s model is still very much there, even on the alternative understanding of the concept of function outlined above.
Another difference between the discussed standpoints concerns the content of a proposition and is related to the problem of its constitution. In Ajdukiewicz’s approach, propositions are constituted by individual objects and properties/relations (making up the set of values of the function identified with a given proposition) and syntactic positions (making up the domain of the function-proposition) – the latter determine the order of the terms in the proposition. Things are different in King’s theory, where syntactic properties belonging to a proposition enter into it indirectly – as constituents of the propositional relation binding the appropriate constituents of the proposition, that is, the appropriate objects and properties/relations. It is worth considering if this difference is not apparent; it could be assumed that syntactic positions play a similar role in Ajdukiewicz’s theory as tree branches do in King’s – that is, they represent the order of the objects in a proposition. It seems that both in Ajdukiewicz’s and in King’s proposal, a proposition is nothing other than the objects this proposition, taken a certain way, is about (Ajdukiewicz 1967/1971, p. 124). On the other hand, it is unlikely that either Ajdukiewicz or King would be inclined to claim that propositions are about syntactic positions or tree branches (or brackets used to reflect the structure of the proposition). This assumption seems legitimate in light of the sameness of the truth conditions generated by both conceptions, as indicated above; it is also perfectly acceptable given the interpretation of Ajdukiewicz’s function-proposition as the occurrence of a certain ordering of designates.

The difference between the two interpretations of Ajdukiewicz’s conception – referring to two different approaches to the concept of function – can appear to be unimportant, even purely terminological. The ease with which the alternative approach has allowed us to nearly equate Ajdukiewicz’s and King’s conceptions might arouse skepticism. According to King, one of the most significant advantages of his conception is that it draws an equivalence between propositions and facts, as opposed to logical constructs. As it turns out, the only step that had to be made in the case of Ajdukiewicz’s conception to move from an identification of a proposition with a formal construct to its identification with a fact was the assumption of an alternative understanding of the concept of function – incidentally, this understanding does not seem to be at odds with the more traditional set-theoretical approach. In light of this, it is an open question if identifying propositions with facts is in fact as significant as King makes it out to be.
3. The Benacerraf Problem

King’s unwillingness to identify propositions with any kind of formal construct stems from his conviction that all conceptions that make such an identification inadvertently fall into the sort of trouble indicated by Paul Benacerraf (1965). The problem arises when one model permits two (or more) equally adequate yet mutually contradictory representations of a given phenomenon. In some theories of propositions, the problem is that the ordered n-tuple identified with a given proposition is determined unequivocally. One can thus conclude that these standpoints face the Benacerraf Problem. In the case of Ajdukiewicz’s theory, no oversight of this gravity can be noted. His method of syntactic analysis is sufficiently determined and based on the fundamental distinction into expressions-operators and expressions-arguments, thus yielding at least apparently unequivocal results.

However, it is not difficult to notice that, using Ajdukiewicz’s method, two different and at the same time intuitively equally good syntactic description of the same sentence can be given. For example:

<table>
<thead>
<tr>
<th>Ajdukiewicz</th>
<th>was</th>
<th>the son-in-law of Twardowski</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>(1, 0)</td>
<td>(1, 2)</td>
</tr>
</tbody>
</table>

Both analyses of the sentence are correct, and it would be difficult to argue for the superiority of one over the other on purely syntactic grounds. It would thus appear that we are facing a typical example of the Benacerraf Problem here. This is not the case, however, since the analyses in question do not constitute two competing representations of the same. The syntactic ambiguity they reveal matches a semantic ambiguity in this case—according to analysis (i), the discussed sentence states Ajdukiewicz’s membership in

\[\text{Desdemona loves Cassio}\] can be represented by any of the following formulas (where L refers to loving): (1) \(\langle\text{Desdemona}^*, L^*, \text{Cassio}^*\rangle\); (2) \(\langle\text{Cassio}^*, L^*, \text{Desdemona}^*\rangle\); (3) \(\langle L^*, \text{Desdemona}^*, \text{Cassio}^*\rangle\); and (4) \(\langle L^*, \text{Cassio}^*, \text{Desdemona}^*\rangle\). The problem here is not only the multiplicity of possibilities but also the fact the same four n-tuples can be considered to represent the proposition expressed by the sentence Cassio loves Desdemona.
Two Models of Propositional Structure

the class of Twardowski’s sons-in-law, and according to analysis (ii), it states that the relation of being a son-in-law obtains between two individuals: Ajdukiewicz and Twardowski. The possibility of giving two different syntactic descriptions of this sentence does not indicate that Ajdukiewicz’s method of syntactic analysis yields ambiguous results but that the sentence can express two different propositions:

\[
\text{(i) } \{((1, 1), \text{Ajdukiewicz}^*), ((1, 0), \text{belonging to}^*), ((1, 2), \text{being Twardowski’s son-in-law}^*)\}^{26}
\]

\[
\text{(ii) } \{((1, 1), \text{Ajdukiewicz}^*), ((1, 0), \text{being a son-in-law of}^*), ((1, 2), \text{Twardowski}^*)\}
\]

It is not the case that in instances of this kind the method of syntactic analysis turns out imprecise and the standpoint faces the Benacerraf Problem. On the contrary, the method allows us to uncover a phenomenon consisting in two token sentences of the same type expressing two different propositions already at the level of syntactic properties. However, there is a certain special group of sentences whose analysis within Ajdukiewicz’s theory does lead to the Benacerraf Problem. They are sentences whose main operator is an expression constituting a two-argument functor referring to some symmetrical relation (regardless of the kind of arguments is might take). Typical examples of such sentences are sentences of the form A=B, or A≠B, in which the main operator is an expression constituting a two-argument functor referring to the symmetrical relation of identity or nonidentity, respectively. The analysis of sentences of this kind can be carried out in a twofold manner: such that the first argument of the operator is A and such that it is B. As a result, we get two alternative approaches to the same proposition – and there are no criteria for the selection of one approach over the other as more adequate.

It seems that the only way to avoid the Benacerraf Problem here is to introduce a conventional rule to the effect that the first appropriate expression occurring in a sentence is to be treated as the first argument of the operator. For example, in a false sentence of the form A=B, the syntactic position (1,1) will be occupied by A. This solution cannot be considered

25 As noted by Tałasiewicz (2003, p. 153), the indicated ambiguity is not a frequent occurrence in the vernacular. However, the distinction into membership in a class and being one of two arguments of a relation turns out significant, for example, in ontological discourse.

26 The object constituting the designate of the expression located at (1,2) here is the extension of the predicate “being Twardowski’s son-in-law”, that is, a certain set.
Two Models of Propositional Structure

satisfactory since it is *ad hoc* – it is difficult to point to an independent rationale for this rule other than the need to avoid the Benacerraf Problem. Moreover, the introduction of such a convention leads to an undesirable outcome: in a sentence of the form B=A, position (1,1) is occupied by B, and thus, the propositions expressed by A=B and B=A must be represented, respectively, as:

- \{((1, 1), A^*), ((1, 0), =^*), ((1, 2), B^*)\}
- \{((1, 1), B^*), ((1, 0), =^*), ((1, 2), A^*)\}

These representations are different, and since it would seem that, in a pair of two syntactically different sentences about the same symmetrical two-argument relation, both sentences express the same proposition, we do face the problem of “surplus” representation indicated by Benacerraf. Does this mean that King is right that standpoints identifying propositions with formal constructs ought to be rejected because of the Benacerraf Problem? The answer is yes and no.

King is wrong when he assumes that moving propositions from the domain of formal constructs to the sphere of facts will eliminate the spectre of the Benacerraf Problem. The difficulty related to sentences about symmetrical relations outlined above occurs both in the interpretation of Ajdukiewicz’s theory according to which a proposition is identified with a function and in the reading according to which a proposition is identical with the fact of the occurrence of a certain assignment (function). In the first case it is not clear which of two functions should be identified with the proposition expressed by a sentence of the form A=B; in the latter case, the fact of the occurrence of which function should be considered as identical with the proposition.

As can easily be seen, King’s theory faces an analogous problem. In his conception, the fact identified with a proposition is that certain objects remain in this or that relation – the relevant relation being determined through appropriate formal constructs. In order to report the proposition-fact expressed by a (true) sentence of the form A=B, we must not only identify objects A and B but also describe the propositional relation obtaining between them. The key fragment of this relation is the relation R reflecting the syntactic properties of the sentence. We therefore get, analogously to Ajdukiewicz’s conception, two alternative approaches to the relation R, and thus, two propositions-facts. One consists in the occurrence of an assignment characterizable as \{K, I, [[A^*][=*[B^*]]]\}; the other, in the occurrence of
In short, it turns out that one can speak of two facts here, each of which can be identified with the proposition expressed by \( A=B \).

King attempts to find a way out of this trap by means of a rather surprising proposition. Namely, he accepts the validity of the claim (deemed problematic earlier in this paper) that a sentence of the form \( A=B \) expresses a different proposition than the sentence of the form \( B=A \). As part of his justification of this thesis, King (2007, p. 95) analyzes the pair of sentences \( 2=1 \) and \( 1=2 \). The fact that King considers these sentences to express different propositions follows from his stance on propositions. In his theory, the aforesaid sentences express propositions structured like this:

- \( \{K, I, [[2^*]=[1^*]]\} \)
- \( \{K, I, [[1^*]=[2^*]]\} \)

These propositions are made up of the same constituents but they differ in structure, that is, their constituents are ordered differently, thus constituting different propositions.

This approach raises doubt since it seems that by uttering the sentence \( 2=1 \) one conveys the same information as one does by uttering \( 1=2 \). Analogously, it is difficult to claim that different content is expressed by uttering "Alec likes tomato soup or Alec likes spinach" and "Alec likes spinach or Alec likes tomato soup" etc. One could thus say that the fact that in King’s conception propositions expressed by such pairs of sentences are considered to be different is a defect. To put it another way, one could challenge King by pointing out that his theory is too fine-grained when it comes to the identification of propositions.

Naturally, King is aware of this and does respond to the challenge. According to him, the conviction that the propositions expressed by \( 1=2 \) and \( 2=1 \) are identical is due to the following principle (P):

\[
\text{(P)} \quad \text{Sentences } p \text{ and } q \text{ express different propositions if there is a context } C \text{ such that } O_p \text{ and } O_q \text{ have different truth values in } C, \text{ } O \text{ being a non-transparent sentential operator (King 2007, p. 96).}
\]

A non-transparent sentential operator is a one-argument sentential functor establishing a non-transparent context, that is, one where – to put it simply – the truth value of a compound sentence \( O_p \) is not a function of the truth value of the subordinate clause \( p \) falling within the scope...
of this operator. Examples of such operators include expressions such as: “necessarily”, “John claims that”, “should be a fact” etc. Since there is no context in which the sentences John claims that $2=1$ and $\text{John claims that } 1=2$ would express propositions characterized by different truth values, it is standardly assumed that $2=1$ and $1=2$ express the same proposition. King (2007, p. 97) holds that the aforecited principle for distinguishing between propositions is incorrect for the following reason:

We can think of the propositions expressed by sentences containing a connective in this set as consisting of a proposition (expressed by the embedded sentence) and a property of propositions (expressed by the connective). This “complex proposition” is true at a circumstance iff the constituent proposition possesses the property in question at the circumstance. From this perspective, the claim that two sentences express the same proposition if the results of embedding them with respect to all propositional connectives have the same truth values in all circumstances (and similarly for all syntactically similar pairs), essentially amounts to the claim that propositions that possess all properties of propositions expressed by English (or natural language) propositional connectives in common at all circumstances of evaluation (and similarly for all syntactically similar pairs) are identical (King 2007, pp. 97–98).

According to King, there is no basis for accepting this thesis. He rejects as incorrect the assumption that natural-language sentential operators express all properties that a proposition can possibly have. Therefore, even if two propositions have the same set of all properties expressed by such operators, the most one can say is that these propositions have a lot in common, not that they are identical. It seems that King’s argumentation can be summarized in the following manner: the notion of the identity of propositions entailed by the principle P is a flawed one since it reduces their identity to the identity of the set of expressible properties (expressible by means of sentential operators). In other words, identity, as it is construed in P, is a contingent property of propositions, and it should be a necessary one.

In light of the above, it would be an error to conclude that the fact that in King’s conception propositions expressed by pairs of sentences about identity are different stems from syntactic differences between these sentences (that is, the fact that expressions located in the position of the first and the second argument are switched). Rather, one should say that the structure
of the relevant propositions is not identical, according to King, and this is reflected in the syntax of the appropriate sentences.

However, there are at least two weak points in King’s argumentation. First, it is legitimate to ask what – if not syntactic analysis – provides grounds for the claim that the propositions expressed by \( 2=1 \) and \( 1=2 \) are different. If we had at our disposal some language-independent means of glimpsing into the structure of a proposition, King’s standpoint would be justified. But since this is beyond our capacity, the only thing we have are our linguistic intuitions, and these – or so it seems – incline us toward the claim that the sentences in question express the same proposition. In short, even if we agree with King that there is no good reason to consider the two sentences in question as expressing the same proposition, King does not offer any solid grounds for claiming that they express different propositions.

Secondly, one might say that King is tendentious in his choice of examples since the sentences \( 2=1 \) and \( 1=2 \) are false (or, alternatively, say something about two different objects). Once true sentences about identity are considered, King is bound to face the problem of doubling the same object in one proposition. For example, the structure of the propositions expressed by:

\[
(CC) \text{ Cicero}=\text{Cicero} \\
(CT) \text{ Cicero}=\text{Tullius}
\]

is the following according to King’s theory:

\[
S(CC) = \{K, I, [[\top]=*[\top]]\} \\
S(CT) = \{K, I, [[\top]=*[\top]]\}
\]

\(27\) The weak “initial” point, noted by an anonymous referee, as mentioned earlier, can be added to this list (I am, again, most obliged for this comment). Namely, the principle \((P)\) is construed as a criterion for stating that two propositions are different, but King interprets it as stronger than it actually is (one might say that he reads an implication as if it were an equivalence) and assumes that if two propositions do not satisfy this criterion, then they are to be considered identical. This misuse shows that his diagnose as to the reason why it is standardly thought that \(1=2\) and \(2=1\) express the same proposition (and not two different propositions) is misplaced – the reason for this lies elsewhere. This in fact invalidates his entire argumentation.

\(28\) In order to avoid confusion stemming from using names in the description of propositional structure, I use a graphic representation of Cicero – after all, according to the discussed conception, Cicero himself enters into the proposition.
In order to avoid the claim that these propositions are identical, King would have to conclude that Cicero featuring in the structure of the proposition \( P(\text{CT}) \) on the left is given in some other manner than Cicero situated in the structure of this proposition on the right. However it is clear that to consider the manner in which an object is given as a part of the proposition is to abandon the conception of direct reference and singular propositions. And this is seriously at odds with the assumptions of King’s theory.

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The above discussion can be briefly summarized as follows: (i) the two theories explored here are based on the same idea that the proposition expressed by a sentence is a fact that the designates of the expressions making up this sentence remain in an order corresponding to the syntactic structure of the sentence; (ii) differences between the discussed theories turn out to be apparent once an alternative (non-set-theoretical) understanding of the concept of function (more specifically, the function identified with a proposition according to Ajdukiewicz) is introduced; (iii) although it seems at first glance that King’s theory is better equipped to cope with the Benacerraf Problem, his argumentation is ultimately unconvincing, prompting the conclusion that neither of the analyzed theories is immune to this problem.

Bibliography


Two Models of Propositional Structure


