Anna Pietryga
SEMIOTICS OF THE DUNS SCOTUS LAW

Originally published as "Semiotyka zasady Dunsa Szkota," Studia Semiotyczne 23 (2001), 129–137. Translated by Klaudyna Michałowicz.

In writing about the semiotics of the Duns Scotus Law (which I shall henceforward refer to as the DSL), I shall begin with a presentation of the law itself and a brief biography of its author, as well as a discussion of those elements of its sign character which, being an expression of the language of logic, it evinces. The remaining part of the article will be devoted to those of its sign features which do not spring directly from the character of the language in which this law has been formulated, but on the contrary, constitute its specific property. That final part of my article can be summarized in the three following theses:

l. the DSL in itself does not unequivocally determine the function of material implication;

2. the DSL expresses the less intuitive aspect of material implication: its truth value when a false antecedent is given;

3. by putting a contradiction in the place of the false antecedent, the DSL presents this contradiction as a model falsehood.

The Duns Scotus Law affirms that from a pair of contradictory statements, accepted in the logical system as its thesis, arises every sentence of that system. Symbolically, it has the following form:

(l) $(p \wedge \sim p) \rightarrow q$ (conjunctive form)

or, equivalently, on the basis of the laws of exportation and importation,

(2) $p \rightarrow (\sim p \rightarrow q)$ (conditional form).

Formulation of this law is ascribed to a Scottish Franciscan named John of Duns,[1] known as Ioannes Duns Scotus, who lived in the late 13^{th}

[1] On the now cleared doubts regarding his place of birth, see Włodarczyk 1988:

and early 14th century (1266[2]–1308). He lived a monastic life since early youth, completing his novitiate in 1280 (Łukaszyk, Bieńkowski, Gryglewicz 1989: 354). He taught, among others, at Cambridge, Oxford, Paris[3] and Köln[4], where he died and where is still venerated today.[5] A philosopher and theologian, honoured with the appellation of the Subtle Doctor (*Doctor Subtilis*) due to the exceptional finesse of his reasoning, in Church history he is remembered as a defender of the doctrine of the Immaculate Conception of the Virgin Mary.[6]

A number of works once attributed to Duns Scotus are now considered to be inauthentic. Among them is the commentary to Aristotle's *Prior Analytics*; its anonymous author is known as the Pseudo-Scotus (Włodarczyk 1955: 2 and 5ff, 1988: XVI). The law of logic discussed in this article can be found in Scotus's authentic writings, although only in its conjunctive form;[7] it seems, however, that it would be more appropriate to ascribe it to Pseudo-Scotus, whose analysis of this law and the related issues is far more thorough (Włodarczyk 1955: 64ff).

Analysing the Duns Scotus Law exclusively as a language sign on the level of some literalness, we may refer to the division of semiotics popularised by Charles Morris and speak of the syntax, semantics and pragmatics of

IX–X.

[2]Or 1265. See Włodarczyk 1988: XI.

[3]From where he was relegated in 1303 for refusing to sign the appeal of King Philip IV of France (Philip the Fair) addressed to the Ecclesiastical Council against Pope Boniface VIII. He soon returned to his post, but left Paris shortly after, probably again for political reasons. See Włodarczyk 1988: XII–XIII.

[4]All four places are mentioned by Włodarczyk (1988). *Encyklopedia katolicka* (Łukaszyk, Bieńkowski, Gryglewicz 1989) gives the exact periods of his stay at Cambridge (1297–1300), Oxford and Paris, overlooking his teaching and research work at Köln. Internet sources with which I am familiar mention a year's period of work in Köln, but are silent regarding Cambridge.

[5]The area of Nola in Italy is another centre of his cult. The process of his beatification was hindered by a rumour that he had been buried alive. This view, now considered groundless, initially caused much jubilation among his adherents. A grotesque 15th-century commentary reads: "This is how sweetly and pleasantly that man passed away from life: from peace to peace, from sweetness to sweetness, from spiritual consolation to eternal joy. May the One who Lives grant the same to us" (after Błoch 1986: 92-93; translated for the purpose of the current article — translator's note). See Łukaszyk, Bieńkowski, Gryglewicz 1989: 354 and Błoch 1986: 87-97.

[6]Officially accepted as the dogma of the Catholic Church only as late as 8th December 1854 (by Pope Pius IX). See Guitton 1966: 342–350.

[7]According to the list of theses of sentential logic in Duns Scotus, found in Włodarczyk 1955: 93ff.

this law. The syntax it uses is, of course, the syntax of the language of logic (i.e. sentential calculus), which is applied consistently; due to this, we are dealing with a meaningful expression. The semantics of the above formula is defined by the general rules of interpreting the expressions of this language;[8] on the basis of these rules it affirms that from a pair of sentences (in the logical sense) which are mutually contradictory arises any sentence which can be formulated in the same language. The pragmatics of the DSL can be defined by its concrete applications.

The above remarks refer to the DSL only as to a formula written in the language of a certain logical system and read in accordance with the rules valid therein. To present the issue of the semiotics of the DSL in an exhaustive manner, it is necessary to mention its other aspects, which are difficult to take account of in Morris's pattern.

There are at least three such aspects; they refer to the DSL's relation to:

1. the function of material implication;
2. the interpretation of this function;
3. the question of the place of contradiction in logical systems.

1. THE DSL'S RELATION TO THE FUNCTION OF MATERIAL IMPLICATION

The currently accepted interpretation of the material implication functor was known already in Antiquity due to Philo of Megara, although it was a matter of some contention (Łukasiewicz 1961: 182-183, Bocheński 1993: 29-30); in fact, also in the writings of Duns Scotus and Pseudo-Scotus some ambiguities related to those contentions are found, but Pseudo-Scotus, as opposed to Duns, attempts to organize and clarify them (Włodarczyk 1955: 19-30). It is worth recalling that both the first axiomatic formulations of logic and the first matrices defining the semantics of truth-value functors appeared in modern Europe only towards the end of the 19^{th} century (Roberts 1973: 131).[9] Earlier, therefore, the role of theorems in interpreting functors appearing therein was more essential.

[8] The question of the character of the relationship between the inscription, being a material substrate of the sign in question, and its meaning is interesting. I have in mind the iconicity of inscription postulated by Peirce; according to this postulate, expressions of logic should be formulated in the form of graphs (Roberts 1973:123ff).

[9] Bocheński remarks that Peirce, to whom the invention of the truth-value matrices is ascribed, "found them in the Megareans"; yet in the same text he uses the example of Philo's *versus* Peirce's definition of material implication to illustrate parallels between various logicians' independent achievements (Bocheński 1993: 33 and 29).

The appreciation of the role of theses proposed by various branches of scholarship in determining the meaning of the signs appearing in those branches (terms, symbols etc.) is due to the French conventionalists, who pointed out the fact that theses of a given field can serve as a substitute for the explicitly formulated definitions. In particular, the axioms of logic may impose certain meanings of given functors.[10]

Given the stipulations of the two-value extensional logic, which postulates the following points:

1. logical functors are (with the exception of one-argument negation) two-argument truth-value functors, unambiguously specifying the course from the two values ascribed to the arguments to the single logical value ascribed to their combination,

2. the system considers two logical values, 0 and 1 (traditionally correspondent to falsehood and truth),

there exist sixteen possible interpretations of two-argument functors, of which two (the *verum* and the *falsum*) are trivial.

Sometimes the interpretation of a functor is determined by a single formula. This is precisely the case of the formula which asserts that truth results from everything (the so-called Law of the Antecedent):

(3) $p \rightarrow (q \rightarrow p)$,

which is a law of logic only if the arrow is interpreted as an implication (or as the *verum*).

The case of the DSL is different. It is easily seen that the implication formula of the DSL, which contains only one two-argument functor, is fulfilled (given the established negation[11]) not only by the material implication (and the *verum*), but also by the alternative. Also in the conjunction form, in which we are dealing with two two-argument functors, establishing the interpretation of one of them in a free manner does not unambiguously determine the interpretation of the other, with the exception of the cases where the imposed interpretation is the *verum.*[12]

Therefore, in neither of the above-mentioned forms does the DSL unambiguously determine the function which would constitute the interpretation of the $\rightarrow$ symbol.

[10] On the related proposal of Hilbert and Bernays, cf. Marciszewski 1987: 18.

[11] It must be added that the implication form of the DSL was used by Hilbert precisely as an axiom of negation (he treated the Law of the Antecedent as one of the axioms of implication); cf. Kolmogorov 1971: 418.

[12] The relationship between the interpretation of both the functors is illustrated by the tables below (numbers in the Table 1 correspond to columns in Table 2):

$(p \oplus \sim p) \otimes q$

2. THE DSL AND THE INTERPRETATION OF THE FUNCTION OF IMPLICATION

The DSL pertains to the least intuitive aspect of the truth-value function connected with implication: its value for formulas with a false antecedent. This issue might be considered to be lying outside the scope of considerations proper to logic, if we assumed that the exclusive domain of this science is the description of laws which can serve as infallible rules of inference and from true premises permit to draw, always and exclusively, true conclusions. Then, a logician would not be obliged to bother with inferences that begin from premises containing a material error; the appropriate truth-value function would only determine that for an implication to be true, not only the antecedent but also the consequent must be true. (The application of the laws of logic does not lead outside the set of true sentences.)

Usually, however, all four initial possibilities are considered in the description of the material implication function, similarly to the other truth-value functions. This approach facilitates generalizations, such as ”truth results from everything” and ”everything results from falsehood”. The Law of the Antecedent expresses the first of those regularities by describing the cases of

---

$\oplus$	$\otimes$
0	3 7 11 15
1	3 7 11 15
2	15
3	15
4	15
5	15
6	3 7 11 15
7	12 13 14 15
8	3 7 11 15
9	3 7 11 15
10	15
11	15
12	7 11 15
13	15
14	12 13 14 15
15	12 13 14 15

p	q	$\times$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	$\times$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
1	0	$\times$	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	1	$\times$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	0	$\times$	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

implication with a true consequent. The DSL reflects the other one, since it pertains to the consideration of all the implication formulas with a false antecedent as true. It therefore expresses the other side, or the other half, of the truth about the interpretation of the implication functor. This is, however, the more bothersome half, inasmuch as it is less compatible with the colloquial understanding of consequence.

This issue has long been raised by various authors. In justification for the logical matrix accepted for the material implication, a number of additional commentaries have been proposed, in the hope of bringing it closer to the general intuition.[13] Peirce compared implication to a not-strict inequality relation between logical values (a comparison especially useful in defining the implication function in multiple-valued logics); an analogous parallel pertains to the set-theory interpretation of the relation of result between (predicated) names (e.g. Keenan, Faltz 1985). A detailed analysis of differences between the colloquially understood conditional and the material implication was presented by Ajdukiewicz (1985),[14] who made the differentiation between what each of these expressions states and what each of them expresses.

Modal interpretations of the deontic type constitute a separate group of commentaries. A sentence of the $p \rightarrow\ q$ type is there "translated" as e.g. "To fulfil action p, you must have the permission q" or "If you do p, you must do q". In particular: "If you made a promise, you are obliged to fulfil it; if you did not make a promise, fulfilment of the given action is morally neutral".[15]

Another explanation can be proposed: when an implication with a false antecedent is formulated, the system of logic is being applied contrary to its purpose, which is to lead from true premises to true conclusions. Assuming

[13] Cf. the catalogue of didactic methods of introducing material implication, with an attempt at classification, in: Clarke 1996. Clarke mentions, among others, Korfhage's interesting interpretation of material implication related to programming languages, although he concurrently notes that this interpretation is not free from error.

[14] According to Ajdukiewicz, both the conditional and the material implication STATE that it is not concurrently so, as the antecedent says and differently than the consequent does; however, the conditional (in contrast to the material implication) additionally EXPRESSES the speaker's lack of knowledge regarding the possible falsehood of the antecedent or truth of the consequent, and his readiness to conduct an appropriate process of drawing a conclusion. The same topic is discussed in the article by Pelc (1986), which emphasises the importance of semantics in the case of implication, and pragmatics in the case of the conditional. For criticism of Ajdukiewicz's viewpoint, see Bogusławski 1986a. Cf. also the polemic of Jadacki and Bogusławski on the same topic, Jadacki 1986, Bogusławski 1986b.

[15] This last observation I owe to Prof. Jerzy Pelc.

every possible sentence to be true, the system is indeed lying; but it is doing this ostentatiously, thereby signalling that from now on it shall not be of much use, because in the given situation it refuses to cooperate.

3. THE DSL AS AN EXPRESSION OF THE LOGICIANS' ATTITUDE TO CONTRADICTION

In Duns Scotus's authentic works only the conjunctive form of his law is found, that is the formula $(p \wedge \sim p) \rightarrow q$.[16] Both its implicative form and any other thesis that would directly express the principle that "everything results from falsehood", are absent. Neither do those works contain a precisely formulated definition of implication (Włodarczyk 1955: 19) or, for that matter, the Law of the Antecedent.[17] This permits us to assume that Duns treated contradiction as a falsehood *par excellence*, by means of which the essential property of implication can be expressed. Pseudo-Scotus, however, making a distinction between the two forms of that law, clearly indicates that in the implicative form conclusion is drawn from a false sentence, whereas in the conjunctive form we are dealing with a conclusion drawn from the impossible (Włodarczyk 1955: 71),[18] and, in the light of this remark, Duns's thesis would probably not apply to falsehood at all. Yet the views of Duns himself permit us a moderate defence of our stance, since he asserts that "everything remains in the same relation to the truth as to existence" (Włodarczyk 1955:44).

Considering contradiction to be a model falsehood (or the model example of the impossible) is in line with a centuries-old tradition in logic, which demanded, and still demands, to unconditionally avoid contradiction.[19] The significance of DSL pertains therefore to several centuries of tradition in

[16]In Włodarczyk (1955) this formula bears the symbol Sz.4,4. All formula symbols below are from that work.

[17]Among the theses formulated by Duns Scotus there is, however, the thesis: $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ (Sz. 2,9.), written also in the conjunctive form (Sz. 4,19.).

[18]Cf. however Pseudo-Scotus's more liberal stance on the same point, *ibidem*, pp. 61-62.

[19]Aristotle's attempts to prove the principle of contradiction are worth mentioning; these attempts, which were futile, are commented upon by Łukasiewicz: "Whoever with great emphasis and self-confidence proclaims a thesis, not giving any proof, whoever IS ANGERED instead of giving argumentation, probably does not have strong enough arguments" (Łukasiewicz 1987: 38; translated for the purpose of the current article — translator's note); the case resembles the former geometricians' inability to abandon the Parallel Axiom.

logic and to its inflexible stance regarding this point.[20] The DSL gains this latter value due to the placement of the contradiction mark in this concrete context.

It needs to be added that the solution applied in the DSL makes it possible to put down the above-discussed property of implication in a symbolic way, and thus to avoid verbal commentaries such as: ”If p is false, any other sentence results from it” (which in fact could not, of course, motivate Duns, who wrote down his theses in Latin). Among Duns's theses there is also a number of other formulas pertaining to the bothersome situation resulting from a false sentence, although in a much narrower sense: there, from a false sentence results only in a contradiction or a sentence earlier determined to be false. The meaning of those formulas can be summarised as ”falsehood results only from falsehood”. Among them are, for example, the following expressions:

(4) $(\sim p \rightarrow q \wedge \sim q) \rightarrow p$ (Sz.4,21.)

(5) $(p \rightarrow p \wedge \sim p) \rightarrow \sim p$ (Sz.4, 16.)

(6) $\sim q \rightarrow [(p \rightarrow q) \rightarrow \sim p]$ (Sz.2,8.)

The above theses are analogical to the proposal put forward by Kolmogorov, which he called the Contradiction Principle:

(7) $(p \rightarrow q) \rightarrow [(p \rightarrow \sim q) \rightarrow \sim p]$ (Kolmogorov 1971: 421)[21]

and similarly to this principle they remain silent regarding the ”unlimited possibilities of drawing conclusions from falsehood”. Their significance pertains more to the *reductio ad absurdum*, which contains, to use Czeżowski's phrase, an essential element of the ”usefulness of error” (Czeżowski 1958).

Acceptance of the DSL means that no pair of contradictory statements, or any other formula with the logical value of 0, can be accepted into the logical system, for it threatens a ”system overfill”: the system becomes trivialised by accepting all the sentences possible to formulate in it as true. This approach excludes the possibility of taking account of, for instance, contradictory data derived from varying sources, in the system of reasoning. The first of the so-called paraconsistent logics[22] were constructed only towards the 1940's; there, the operation of the DSL is limited in various

[20]Limited only to the created world by some thinkers e.g. Pietro Damiani or Nicholas of Cusa, a point with which Duns Scotus clearly disagreed; see Włodarczyk 1955: 44-45, Nicholas of Cusa 1997.

[21]Kolmogorov presents this formula as a version of the Contradiction Principle possible to accept as an axiom in intuitionistic logic. This formula was indeed included in the list of axioms given by Heyting (Kolmogorov's text was written in 1925).

[22]See the pioneering work by Jaśkowski (1948).

ways,[23] which makes it possible to accept contradictory theses with no risk of the system exploding.

Bibliography

1. Ajdukiewicz, Kazimierz (1985) "Okres warunkowy a implikacja materialna." In *Język i poznanie* II, 248-265. Warsaw: PWN.

2. Błoch, Grzegorz B. (1986) *Jan Duns ze Szkocji*. Poznań: Franciszkanie.

3. Bocheński, Józef Maria (1993) "Rozwój logiki formalnej." In *Logika i filozofia*, 22-34. Warsaw: PWN.

4. Bogusławski, Andrzej (1986a) "Analiza zdań warunkowych a problem funkcji semiotycznych." In *Studia semiotyczne*, ed. Jerzy Pelc, XIV-XV: 215-224. Wrocław: Ossolineum.

5. Bogusławski, Andrzej (1986b) "Jeszcze o Ajdukiewicza koncepcji őwyrażaniaŕ." In *Studia semiotyczne*, ed. Jerzy Pelc, XIV-XV: 249-270. Wrocław: Ossolineum.

6. Clarke, Matthew C. (1996) "A Comparison of Techniques for Introducing Material Implication". http://simon.cs.comell.edu/lnfo/People/gries/symposium/darke.htm.

7. Czeżowski, Tadeusz (1958) "Pożyteczność błędu". In *Odczyty filozoficzne*, 301-304. Toruń: Towarzystwo Naukowe w Toruniu.

8. Łukaszyk, Romuald, Bieńkowski Ludomir, Gryglewicz Feliks, eds. (1989) *Encyklopedia katolicka*, Vol. IV. Lublin: Towarzystwo Naukowe KUL.

9. Guitton, Jean (1966) *Jezus. Maryja*. Warsaw: PAX.

10. Jadacki, Jacek Juliusz (1986) "O zdaniach warunkowych." In *Studia semiotyczne*, ed. Jerzy Pelc, XIV-XV: 225-247. Wrocław: Ossolineum.

11. Jaśkowski, Stanisław (1948) "Rachunek zdań dla systemów dedukcyjnych sprzecznych". Studia Societatis Scientiarum Toruniensis I[5]: 57-77.

[23]For the classification of paraconsistent logics, see Priest, Routley 1984 and Priest 1988.

12. Keenan, Edward L., Faltz Leonard M. (1985) *Boolean semantics for natural language*. Dordrecht: D. Reidel Publishing Company.

13. Kolmogorov, Andrey Nikolaevich (1971) ”On the principle of excluded middle.” In *From Frege to Godel. A source book in mathematical logic. 1879-1931*, ed. Van Heijenoort Jean, 414-437. Cambridge (Mass.): Harvard University Press.

14. Łukasiewicz, Jan (1961) ”Z historii logiki zdań”. In *Z zagadnień logiki i filozofii*, 178-194. Warsaw: PWN.

15. Łukasiewicz, Jan (1987) *O zasadzie sprzeczności u Arystotelesa*. Warsaw: PWN.

16. Marciszewski, Witold, ed. (1987) *Logika formalna*. Warsaw: PWN.

17. Nicholas of Cusa (1997) *O oświeconej niewiedzy*. Cracow: Wydawnictwo Znak.

18. Pelc, Jerzy (1986) ”Jeżeli, to.” *Studia semiotyczne*, ed. Jerzy Pelc, XIV-XV: 271-286. Wrocław: Ossolineum.

19. Priest, Graham and Richard Routley (1984) ”Introduction: paraconsistent logics.” *Studia Logica* 43[1-2]: 3-16.

20. Priest, Graham (1988) ”When inconsistency is inescapable: survey of paraconsistent logics.” *South African Journal of Philosophy* 7(2): 83-89.

21. Roberts, Don D. (1973) *The Existential Graphs of Charles S. Peirce*. The Hague–Paris: Mouton.

22. Włodarczyk, Tadeusz (1955) *Logika zdań u Jana Dunsa Scotusa i Pseudo-Scotusa*. Łódź (typescript of doctoral thesis).

23. Włodarczyk, Tadeusz (1988) ”Wstęp.” In *Jan Duns Scotus, Traktat o Pierwszej Zasadzie*, VII-XXI.