Urszula Żegleń

ONTOLOGICAL ISSUES IN NON-CLASSICAL LOGIC

Originally published as "Problematyka ontologiczna logik nieklasycznych," Studia Semiotyczne 16–17 (1990), 217–249. Translated by Wojciech Wciórka.

Abstract
The aim of the paper is a brief overlook at some philosophical issues of non-classical logic, and more strictly – modal logic, included also interpreted modalities in epistemic, temporal and deontic logic. I begin with some questions asked in philosophy of logic in reference to modal logic, especially in regards to its semantics given by Saul Kripke with application of the ontological notion of "possible world". This notion will be in the centre of considerations. In the first chapter I shall make short remarks on Kripke's model and on the characterisation of the relation between possible worlds. I shall point at the main approaches to possible worlds in philosophy of logic and some ontological issues. In the second chapter I shall focus on interpreted modalities, successively in epistemic, temporal and deontic logic. I shall be interested in replying to the question "what kind of ontology (with what kinds of objects) is implied by each of these types of logic?" .

Key words: modal logic, modality, interpreted modality, possible worlds, accessibility relation.

1 Ontological issues in modal logic

The rise of non-classical logics was a turning point in the history of logic: it set the stage for new metalogical research and shed new light on the philosophical problems involved in logic. Still, also the classical logic already entails certain philosophical presumptions, even though its

origin is closer to the foundations of mathematics than to philosophy. The Platonist or nominalist assumptions made in philosophy affect the choice of language, which either accepts or rejects general objects in its semantics; yet another assumptions, made from the position of realism or particularism, determine the character of the primitive terms, which correspond to qualities or individuals in semantics.[1]

Philosophical issues (especially those about existence) involve quantification of linguistic expressions; certain philosophical preconceptions also form the basis for discussions concerning three basic semiotic functions: naming, denoting, and meaning. For philosopher especially rich in ontological assumptions and implications is non-classical logic which includes systems in which philosophical (e.g. modal or epistemic) notions are formalized, they are the most ontologically committed systems.[2]

Many non-classic logics are based on modal logic, whose ontological commitment is imposed on it by the possible worlds semantics. Contemporary logicians adopt philosophical interpretations of possible worlds in the hope that the domains determined by logical models in some sense correspond to the ontological universe examined by philosophers. Yet this gives rise to questions such as: what is a possible world? In what way does it exist? What is the difference between the actual world and a merely possible one? What relation do they bear to each other? What does it mean that a state of affairs exists in every possible world? Etc. As a result, logicians, or rather philosophers of logic, who raise these questions, revive the aged ideas of Leibniz, Kant, and Meinong. Let us consider these ideas by drawing on the now classical Kripke's semantics, in order to realize the depth of semantically induced ontological commitment of modal logic.

Kripke's semantics for normal modal logic consists in the well-known model $<K, G, R>$, where K is a non-empty set (informally defined as a set of possible worlds), G is a distinguished element of K (interpreted as the "real". i.e. actual world), R is a relation between the worlds, called the "accessibility (or "possible relative") relation" (Kripke 1963: 68f). In one of the most familiar systems of normal modal logic,[3] i.e. Feys' system T, the R

[1]There is a vast literature on this topic in which the classical position is already Nelson Goodman's monography *The Structure of Appearance* (Goodman 1977).

[2]The concept of ontological commitment has been introduced to the contemporary philosophy of logic by Quine (1948) and Church (1958), in order to refer to ontological assumptions and philosophical issues entailed by them, especially the problem of existence in logic.

[3]The normal modal logic among its systems includes such known systems as: Feys' system T and C.I. Lewis's systems S4 and S5. The name "T" for Feys' system (con-

relation is reflexive, namely: for any w in K, wRw, which means that any world w is accessible from itself. Let α be short for the expression $\mathrm{L}p \rightarrow p$ (which is the axiom of necessity accepted in the system T). Assume that $\mathrm{V}(\alpha, \mathrm{w}) = 0$. That is the case if and only if $\mathrm{V}(\mathrm{L}p, w) = 1$ and $\mathrm{V}(p, w) = 0$. $\mathrm{V}(\mathrm{L}p, w) = 1$ just in case the sentence p describes a state of affairs which occurs in each possible world accessible from w, and $\mathrm{V}(p, w) = 0$ just in case p describes a state of affairs that does not take place at w. In order to guarantee the truth of α, it is enough to assume that each world w is accessible for itself. If at w there are some rational creatures, then it is plausible to believe that they know the state of affairs occurring in their own world. Thus accessibility may be intuitively construed as a certain cognitive ability pertaining to the inhabitants of a given world.

In a stronger system of normal modal logic, i.e. in S4, the accessibility relation R receives in addition the property of transitivity, namely: for any $w_i, w_j, w_k \in K$, $w_1Rw_2 \wedge w_2Rw_3 \rightarrow w_1Rw_3$. Let us assume that R is not transitive. This means that there are w_1, w_2, w_3such that w_1Rw_2 and w_2Rw_3, but $w_1 Rw_3$. Then taking into account the axiom $\mathrm{L}p \rightarrow \mathrm{LL}p$ from S4 we can consider the valuation V such that $\mathrm{V}(\mathrm{L}p. w_1) = 1$ and $\mathrm{V}(p, w_3) = 0$, thus $\mathrm{V}(\mathrm{L}p, w_2) = 0$ and $\mathrm{V}(\mathrm{LL}p, w_1) = 0$. This means that the axiom $\mathrm{L}p \rightarrow \mathrm{LL}p$ has been falsified. Thus, if the axiom $\mathrm{L}p \rightarrow \mathrm{LL}p$ is valid in a frame, then its accessible relation is transitive.

The lines of reasoning presented above are based on the following description of a necessary sentence:

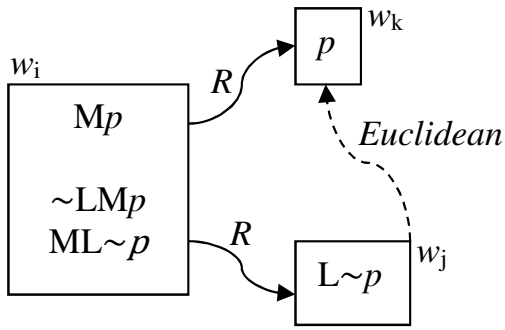
$V(\mathrm{L}p,w_i) = 1 \leftrightarrow \forall w_j \in K\ (w_iRw_j \rightarrow V(\alpha,w_j) = 1)$

Let us now illustrate the relation of transitivity with the following example: $w\backslash_1$ – the actual world, w_2 – the word in 1944, w_3 – the world in 1914; "w_iRw_j" means that the world w_i is accessible from the world w_j. In our example, the world in 1944 is accessible from ours (known from historical materials). Likewise, w_2Rw_3, i.e. the world in 1914 is accessible from the world in 1944, that is to say, in 1944 there existed materials concerning the events of 1914. Consequently, since we can access the world in 1944 and the materials from that period also concern the events of 1914, we can access the world in 1914 as well. (Although for philosopher a situation can be more

structed in 1937) has been given by Bolesław Sobociński. Sobociński gave also the proof for the equivalence of T with the system M of Georg Henrik von Wright (Sobociński 1953). Hence in the literature T is often treated as Feys'-von Wright's system (as for instance in Kripke (1963) where it is said to be the system M(T) of Feys-von Wright). On the connections among the systems of modal logic and its history see: (Hughes, Cresswell 1972, a new completely re-written edition 1996, Latinov 2014) among others.

complicated and deserve more attention).

The accessibility relation becomes symmetric in the model of S5, where we have: $w_iRw_j \rightarrow w_jRw_i$. Symmetry of R follows from Brouwer's axiom: $p \rightarrow \mathrm{LM}p$ which on the other hand can be obtained by the axiom $\mathrm{M}p \rightarrow \mathrm{LM}p$, and the axiom $p \rightarrow \mathrm{M}p$ (which is obtained by T and also belongs to S5). If R would not be symmetric, that would mean that there would exist such worlds $w_1, w_2 \in K$ that w_1Rw_2, while w_2Rw_1. Then we could define such a valuation V that $\mathrm{V}(p, w_1) = 1$ and for each w (if any) would be accessible from w_2 we would have $\mathrm{V}(p, w) = 0$. This would mean that $\mathrm{V}(\mathrm{M}p, \mathrm{w}_2) = 0$, and then it is easily to see that $\mathrm{V}(\mathrm{LM}p, w_1) = 0$. Thus $\mathrm{V}((p \rightarrow \mathrm{LM}p), w_1) = 0$. Coming back to the axiom $\mathrm{M}p \rightarrow \mathrm{LM}p$, it is known that it corresponds to the Euclidean condition: for any $w_i, w_j, w_k \in K$ $((w_iRw_j \wedge w_iRw_k) \rightarrow w_jRw_k)$. Similarly as in the case of the symmetricity condition, it is seen that if R were not Euclidean, then $\mathrm{M}p \rightarrow \mathrm{LM}p$ could be falsified (cf the diagram[4]).



The following description of the truth value of a possible sentence has been employed above:

$V(\mathrm{M}p, w_i)=1 \leftrightarrow \exists w_j \in K(w_iRw_j \wedge V(\alpha, w_j)=1)$

Therefore, it should be accepted that if a world w_j is accessible from the actual world w_i, then also w_i is accessible from w_j. Let w_i be the actual world, i.e. the world of the occurring states of affairs, and w_j – a world from the near past. In that case, if the past world is accessible for a person living

[4]Here I address my special thanks to Wojciech Wciórka who commented this fragment and added the diagram. In describing the properties of the relation R I had the benefit from the comments of Marek Nasieniewski from the Chair of Logic at the Nicolas Copernicus University.

in the actual world (e.g. by virtue of recollection), then, if the person lived in the past at w_j, she was also able to access w_i (e.g. thanks to the ability to predict, forecast, envisage).

The examples given here might be debatable, since they have already been interpreted in a certain way, whereas the formal notation only allows to decipher formal properties of the relation R in various systems. The intuitive construal of accessibility as a certain cognitive capacity makes us consider R pragmatically and apply an epistemic interpretation to modal systems. If in turn the relation is conceived, say, temporally, then modal logic will become the basis for temporal logic. Our understanding of accessibility relation, therefore, determines the specific interpretation of modal logic: epistemic, temporal, deontic, topological, or yet another. Merely providing a model, however, is not sufficient to characterize modality, or even to answer the question of what modality is. Following Alvin Plantinga I repeat his view, according to which the model-theoretic structure is a pure set of theoretical constructions, with no connection to modal terms (Plantinga 1974: 126). A similar outlook has been presented by Marian Przełęcki (1974), who notes the insufficiency of model-theoretic devices for analyzing the philosophical problems implied by modal systems. Thus a new avenue opens up for research necessary for identifying objects occurring in Kripke's model. Various interpretations of modal logic will be left out here, but some accounts of possible worlds will be outlined, with focus on their nature and ontological status. In other words, the presentation of the most popular theories of possible worlds in contemporary philosophy of logic will serve as a means to ontologically characterize the domains relevant to modal systems.

The concept of possible world has a long philosophical tradition, dating back to Plato and revived by Leibniz (in his metaphysics). In contemporary philosophy of logic the following main approaches has been distinguished: (1) linguistic, (2) object-oriented, and (3) epistemic.

1. *The linguistic approach.* It is the most popular one which has been started by Rudolf Carnap (1946, 1947) in his attempt at solving the problems of analyticity, meaning and modality stated by W.V.O. Quine. In his earlier account "a state-description is a class of sentences which represents a possible specific state of affairs by giving a complete description of the universe of individuals with respect to all properties and relations designed by predicates in the system" (Carnap 1946: 50). The states-descriptions (which are bound to the language) are taken to represent possible worlds (although in Carnap's approach this notion does not imply any ontological issues). Treating possible worlds as sets of linguistic constructions is useful chiefly for logical semantics.

Models for modal systems are linguistic constructions in which modality has been connected to truth value. The distinguished value is truth, so the distinguished world is the actual world or the actual state of affairs constituted by a set of true sentences. A valuation function which has been added to Kripke's frame $<K, R>$ has as assignment to determine truth-values (1 or 0) to an atomic formula α in a considered world w. The central idea of Kripke's semantics is that a formula of the form Lα (necessary that α) is true at a world w that means that it is true in all worlds accessible from w through the relation R. While a formula Mα (possible that α) is true at a world w that means that it is true in some worlds accessible from w through the relation R.

2. *The object-oriented approach*. This is a very differentiated approach according to which possible worlds are identified with states of affairs (by which properties of things are conceived) or with total ways things could be (different versions are given in Plantinga 1974, 1976, Stalnaker 1979, Lewis 1979). On this view, modality is a quality of things and as such it is modality *de re*.[5]

3. *The epistemic approach*. Possible worlds are certain possible cognitive situations or objects of intellectual processes (e.g. Hintikka 1962, 1974, Rescher 1974, 1975). A possible state of affairs can be also replaced with certain possible conceptual functions.

Some analogies can be drawn between these approaches and the accounts of universals; thus the interpretation of possible worlds as linguistic expressions is regarded as nominalist, the object-oriented interpretation – as realist, and the epistemic one – as conceptualist. Being aware of the status of possible worlds seems extremely important and probably crucial for understanding the nature of modality. Endorsing the first, linguistic standpoint only allows one to speak of modalities inherent in language. At best, the approach might be extended to various kinds of language described in the philosophy of language, so that one could analyze modal utterances occurring in them; e.g. in regard to natural language, it is possible to consider grammatical forms of the Polish language or use the known semantic analyzes of English;[6]

[5]Modality *de re* is regarded as an attribute of things and is distinguished from modality *de dicto*, an attribute of sentences (or propositions). These two kinds of modality have been introduced in the 12th century by Abelard, and then analyzed by St. Thomas Aquinas in his *Summa contra gentiles*. In this place I would like again to thank Wojciech Wciórka for paying my attention on Abelard's contribution to this issue.

[6]Scholars have discussed, inter alia, English constructions such as "possibly-so-and-so", "this is a possible world for A to x". See Hacking 1975.

regarding particular languages – to analyze prescriptive utterances occurring in the language of ethics or law;[7] with respect to artificial languages – to interpret theorems occurring in modal, epistemic, deontic, or other systems. It is believed, however, that on the linguistic account, modality can only be described as an operator, which could be indexically characterized in various ways, depending on the type of utterance.

Yet the semiotic analysis by itself already opens up new, broader avenues for further study. For instance, the analysis of prescriptive sentences (formalized in deontic logic and playing a normative role) would be fairly shallow if it was limited to characterizing syntactic properties of the deontic operator and did not take into account pragmatic (chiefly normative and also epistemic) aspects of utterances. If in turn, modality is granted an objective status (as *de re* modality), we clearly encroach on ontological ground, which immediately gives rise to a question about the role of modality in ontology. The answer to this question requires a definite theory of an object, in which the modes of existence of different kinds of objects which fulfil its domain, their structure (extrinsic and intrinsic), relations among them is specified. The epistemic approach, in turn, requires acquaintance with philosophical epistemology.

Taking into account the accounts of possible worlds mentioned above, and the problems posed by them, one may wonder if Kripke's model – presented in such a general way and accepted both for modal systems and their various interpretations – could be modified, e.g. into the following forms:

1. for the linguistic approaches:

$<C, C_i, R>$,

only with languages of deductive systems and the relation R conceived as inferential entailment;

2. for the object-oriented approach:

$<K, S, R>$,

where K is one's knowledge, S – objective states of affairs, and R is a relation of intentional correspondence.

3. for the epistemic approach:

$<G, L, R>$,

where G – intentional states of affairs, L – language, R – also a relation of certain intentional correspondence.

[7] These issues have mainly been discussed by analytic philosophers; see the examples of classical positions in question: Black 1949, Hare 1952, Searle 1964.

Obtaining such models, however, would have to be preceded by a precise epistemic interpretation of modal systems. What is important in the accounts of possible worlds selected here is the distinction between objects filling up these worlds, i.e. whether they are constructs of language, of mind, or objective states of affairs. Some hold that there exist systems of actual objects (so called systems of surrogate-worlds), which are structurally isomorphic or analogous to the systems of possible worlds. In the logico-philosophical literature one can encounter the following examples of such systems:

(1) A system of states of affairs understood linguistically, or of Carnap's state-descriptions, characteristic of the linguistic approach and most popular in logical semantics. Here, possibility amounts to consistency, and actuality is interpreted as the value of truth. Such treatment of modality has been criticised by, among others, David Lewis (1968), who notices a vicious circle in Carnap's metalinguistic exposition and thereby points out the impossibility of defining consistency in terms of possibility. Yet besides the syntactic conception of states of affairs, one can also encounter a pragmatic account in the philosophy of logic (Chisholm 1979), where a state of affairs is defined in the following way: p is a state of affairs if and only if p is such that it is possible that there is someone who accepts p. The definition of a state of affairs as an object of acceptance rejects impossible states of affairs (which are not acceptable by any rational subject) and links existing states to a subject. If a subject accepts a state of affairs, then there must be a criterion for this acceptance. If this analysis is applied to formal systems, then the rules for acceptance correspond to rules of inference. Chisholm, however, was more interested in our real cognitive practice which not always is in agreement with ideal requirements of formal logic. In his analysis the concept of acceptance was strictly connected with the concept of rationality and his hierarchy of epistemic values whose list were enriched in sequent three editions of his *Theory of Knowledge* (1966, 1977, 1989).

One may either discuss the adequacy of Chisholm's definition or follow his general intuition that we only deal with states of affairs that are given in cognition, which thereby can be expressed in language in propositions, and only these propositions are to be accepted by a subject. Also possible worlds, due to their relation to subjects, are connected with the intentional realm. If they are placed in the intentional realm, then the states of affairs or objects filling up those worlds appear as objects of thought, whether in the form of propositions or concepts. In that case we may have to deal with false propositions or concepts referring to nonexistent objects.[8] Consequently, should

[8]In logical semantics, the problem of nonexistent objects is considered e.g. with

the possible worlds not be interpreted in the framework of a suitable philosophical conception of intentionality? For Chisholm, however, propositions which are treated as the subclass of the class of states of affairs, the same like them belong to the ideal realm. But with regard to his internalist approach to traditional epistemological issues (especially in question of content and justification, and his view on certainty as the highest epistemic value rooted in selfconsciousness) it seems reasonable to appeal also to the intentional realm.[9]

Among other examples of surrogate worlds the following are worth mentioning:

(2) A system of things conceived in a certain (here: possible) mode in which we as human subjects represent our world (Stalnaker 1979) or a system of maximal states of affairs (Plantinga 1974, 1976).

(3) A system of combinatorial constructs (Quine 1968, Creswell 1972, Armstrong 1989).

(4) A system of Meinongian objects formally rendered by Terence Parsons (1974, 1978), Hector-Neri Castañeda (1974), and Nino Cocchiarella (1982).

All these systems of surrogate worlds are characteristic of the object-oriented account, although the last one might be taken to be better suited for the epistemic approach. What speaks in favour of the object-oriented account is the content-like treatment of Meinongian objects as certain sets of properties and a realist interpretation of Meinong's theory.[10] On the other hand, the epistemic approach might be suggested by the fact that the Meinongian objects are conceived as objects of some mental states.

(5) The epistemic approach is associated with the above-mentioned system of mental constructs, made up of intentional states of affairs or intentional objects (Rescher 1974).

The list of systems of surrogate worlds presented here is certainly neither

respect to a theory of proper names, empty names, meaning, and denoting. See e.g. Kripke 1972, Lewis 1978, Munitz 1974, Pelc 1983.

[9]Chisholm's epistemology and epistemic logic (which in fact is an analysis of basic epistemic concepts without their formalisation) was the subject of many interesting criical discussions. In Polish literature an interesting critical overview of Chisholms epistemology is given by Renata Ziemińska (1998) who is also the interpretator of his works into Polish language.

[10]The discussion concerning ontological status of nonexistent objects produces the same views as the problem of universals, i.e. (i) nominalism (here, originating in Russell and Quine), (ii) conceptualism (dating back to the Stoics and some scholastics, endorsed in different forms by Descartes, Leibniz, and Brentano), (iii) realism (having its source in Arabic philosophy and most explicit in Meinong's theory).

exhaustive nor mutually exclusive. Apparently, however, it is sufficient for our purposes to take into account only the three approaches to possible worlds put forward at the beginning, and thereby maintain that worlds are filled by: (i) linguistic objects (descriptions or sentences) characteristic of the linguistic account, (ii) intentional objects (such as concepts or propositions) characteristic of the epistemic account. Yet it is debatable what kind of objects should be ascribed to the object-oriented account: intentional, real, or maybe ideal?

Again, one can see the need to carry out further analyzes in philosophy, not in semantics. Meanwhile it is worth mentioning other issues discussed in the possible worlds semantics. One of them is the relationship between the actual and the possible worlds. Various resolutions of this problem have been offered depending on the accepted division of worlds, i.e. (A) on account of unity or contrariety of worlds two standpoints are distinguished: (1) one which treats all worlds equally, including the actual world; (2) one which distinguishes merely possible worlds from the actual world, which is supposed to have a distinctive ontological status; (B) on account of whether we underscore actuality or modality, we obtain actualism in a *de re* (Plantinga 1974, 1976, Stalnaker 1979) or *de dicto* version (Adams 1979), and the radical possibilism or in other words – modal realism, called sometimes "hyperrealism" (Lewis 1979, 1986) .

According to modal actualism, each possible world exists as actual (Platonism *sui generis*), although only one of them is really actual, which is understood in various ways inside this general view. Actuality is regarded as a state of world; for Plantinga, it is being momentary, for Stalnaker it is the state of affairs exemplified by the concrete world, i.e. the world we live in. "Being momentary" denotes here a quality pertaining to things, relations, states of affairs. Especially interesting and rich philosophical domain which could be taken into consideration here, is the domain of real world described in the existential ontology of Roman Ingarden in his *Controversy over the Existence of the World.* [11] On the other hand, in the *de dicto* version of actualism, where possible worlds amount to theoretical constructions from linguistic expressions, actuality corresponds to the value of truth. Possible worlds are treated as merely possible world-sentences, while the actual world is a world-sentence possessing the value of truth.

The problem of actuality is presented in a different light by the radical possibilism. All worlds (including the actual world) are possible worlds, and

[11] I made such an analysis in another place, cf. Żegleń 1990: Ch. 5. Ingarden's conception of modality. 2. Empirical possibilities (in Polish).

all possible worlds are equally real; all objects exist in them in an equally real way, and each possibility will be realized in them (radical realism). Intuitively "possible" in reference to a world means a "way that a world could possibly be", i.e. "a way that some world *is*". (cf. Lewis 1986: 7). Among possible worlds, one is distinguished as actual, but not because it differs from others but because we are its inhabitants. Possible worlds are similar to ours, since they are inhabited by equally concrete, spatiotemporally determinate objects. Still, as the inhabitants of our world we have certain non-relative characteristics not possessed by objects in other worlds. In David Lewis's account the very term "actual" does not express any absolute property which would distinguish a given world from merely possible ones, but it is an indexical label, fulfilling the same linguistic function as indexical words such as "here" or "now". "Actual" is understood as "each world w is itself the world w". "Actual world" is therefore synonymous to "this world", which means that each world is actual in itself, and the inhabitants of other worlds can also describe their own world as actual. Worlds are isolated and there are no spatiotemporal relation between objects which belong to different worlds" (Lewis 1986: 7). This means that members of each world are closed in the spatiotemporal border of their own world and they are spatially and temporally connected only within one world.

Lewis's solutions enter deep into existential ontology and give rise to philosophical questions which open a new range of problems which cannot be solved by Lewis's theory itself. First of all, the exposition of his theory in his early works required better clarification. What does it mean that all possible worlds are equally real, that all objects exist in them in an equally real manner, and that every possibility will be realized in them? Surely, it is not the kind of existence scrutinized by metaphysics. Is it then legitimate to say that each world in itself is actual and autonomous – and only in this sense will each possibility be realized in some world? Lewis's primary interest is not, however, any existential ontology (in a strict philosophical sense), but an exact analysis of the logical space with regards to possibilia, especially those which under certain conditions make up the worlds. His possibilism is different from many classical versions of philosophical possibilism (or realism), though with regards to its modal extensionalism. His analyses are metalogical, nevertheless they are some means in doing analytical metaphysics, here metaphysics of modality (cf. Lewis 1986: 17).

An approach clearly distinguishing the actual world from possible worlds is proposed by the above-mentioned combinatorialism. The possible worlds

different from the actual world can be construed as different combinations of the entities filling up the actual world. The alternative worlds are sets of n ordered pairs which determine spatiotemporal location of various particles. Yet even here many ambiguities arise, e.g. how are these particles to be understood: are they independent beings or rather elements (in some sense) of objects occurring in the actual world?

Furthermore, a problem has been posed whether one individual could exist in more than one world. This difficulty, raised by Chisholm (1967) is connected to the problem of transworld identity. It is objected, however, that the concept of transworld individual is (i) contrary to the principle of identity, according to which, if two arbitrary individuals are identical, then they share all properties; (ii) it denies the transitivity of identity. What has also been underlined is the difficulty caused by the lack of criteria for identity of possible objects; Quine's (1948) question is well-known: is the possible fat man in this door identical to the possible bald man in the same door, or are they two separate men? And even if an individual is fixed in a world w on account of some characteristics, there is still no guarantee that the same properties are not possessed by another individual in this world.

For this reason, some regard this problem as meaningless. There are different solutions to this problem. One of them can be found in Lewis's (1968, 1971, 1986) counterpart theory. Identity has been replaced in it by the counterpart relation. He rejected transworld identity just in favour of his counterpart relation. Each particular is limited by a world, and an object existing in one world has its counterparts in different worldsStill, there remains the problem of finding adequate criteria of similarity.[12] Lewis's theory is charged for being contrary to common intuitions about modal notions, although it seems to need a stronger philosophical foundation if it concerns ontological issues.

Yet another difficulty arises in connection to the choice of possible worlds which one wishes to consider in a model. On what basis should one pick out from an ontological universe the worlds which will be considered philosophically interesting? The accounts of possible worlds in philosophy of logic, briefly presented above, do not suffice in this regard: a definite philosophical ontology is needed in order to examine their ontological status, i.e. the mode of existence, to determine if they are independent or not, whether their existence is objective or anchored in a subject (in mind). Again, locating possible worlds in God's mind – as shown by Leibniz – raises

[12]Kit Fine in his critical notes to Lewis' *Counterfactuals* arose difficulties connected with Lewis' notion of similarity (Fine 1975).

a new philosophical question, known in the philosophy of logic as the Leibniz paradox, i.e. the question of choosing the best possible world, which was to be identified with the actual world. In the philosophy of logic, it is analysed and solved with semantic methods, by indicating the antinomial character of Leibniz's statement (cf. Adams 1972, Blumenfeld 1972, 1975, Plantinga 1973, Burkhardt 1980).

Certainly, the problems mentioned above do not exhaust all ontological issues involved in modal logic. Only those most frequently discussed in contemporary logico-philosophical literature have been touched on. For instance, a relatively little known interpretation of particular theorems of modal logic has been omitted, although they also determine certain ontology, in fact, a fairly rich one. Theorems of S5, regarded by many logicians as the most philosophically attractive modal system, seem to be of particular philosophical interest. For example, its theses could be ontologically interpreted and compared to known philosophical claims. The very axiom of S5, $Mp \rightarrow LMp$, seems interesting enough, as well as the similar Brouwer's axiom $p \rightarrow LMp$, whose addition to T produces the Brouwerian system (and added to S4 together with a special rule gives S5).[13]

Thus, again, a new avenue for philosophical research opens up. Should one not, therefore, present a definite conception of object (it is believed that the best suited one would be essentialist) and search it for solutions of the problems described here? Would a phenomenological Ingarden's ontology, an *a priori* theory of pure possibilities, not be the most adequate one? Kripke's model could be then compared with a model, reconstructed in this ontology, which defines the domain of its research. Furthermore, it is believed that acquaintance with domains of the actual world and a possible world would enable us to answer the question asked here: what is modality? The direction of research, which has been chosen here, seems to indicate that the analyzes initiated in formal logic lead up straight to ontology, at first to a formal one (rather in the sense of semantics), and then to a philosophical theory of object (or rather – a theory of possibilities). Does it not mean that the modality formalized at least in some systems in logic is the same modality that is revealed by a philosopher studying the structure of being (or reality)? The present article does not solve this problem, but pointing on the philosophical problems of modal logic is always inspiring for further investigations, which could lead to some more specific ontology implied by modal logic. It does not mean that the proposal of research headed towards

[13]The Brouwer's axiom has been introduced and analyzed by Oscar Becker (1930). More on this issue in a formal aspect see in (Hughes, Cresswell 1972, Ch. III).

phenomenological ontology is the only one possible, but it has been put forward with regard to the essentialist account of this ontology which favours such research (although it should be taken into account that the notion of "essentialism" can be quite differently understood in phenomenological and analytical ontology).

2 Ontological implications of non-classical logics. Metalogical notes on epistemic, temporal, and deontic modality[14(]

Contemporary logic is a philosophically interesting subject for metalogical studies, chiefly thanks to its ontological implications and applicability to the analysis of natural language. Those studies emphasize the ontological significance of non-classical logics, whose creators drew inspiration from philosophy and metalogic (studies on foundations of mathematics). Modal logic is the fundamental part of non-classical logic; ontological implications of its theorems, and above all its possible worlds semantics, raise a lot of ontological issues. The modal concepts themselves are philosophical notions of rich intensional character. They have been given various meanings since ancient times; thus it is common to model epistemic notions ("to know", "to believe", "to be convinced", etc.), temporal ones ("it has always been / will always be the case", "it has been / will be the case", etc.), and deontic ones ("to command", "to forbid", "to obligate", "to permit", etc.) on the central modal notions ("necessary", "possible", "contingent", and their negations), called alethic modalities.

These concepts are counted among modal ones, because, as Hintikka (1963: 151) puts it, they can be analyzed in the same way as the normal, alethic modalities. It is, therefore, philosophically interesting to show these modalities in respect of their philosophical character, that is to say, the ontological problems implied by various types of modal systems, which are constructed by using the following methods: (1) as interpretations of Clarence Irving Lewis' systems of alethic modalities (e.g. Rescher's epistemic systems); (2) as axiomatized systems whose language is defined independently from Lewis's systems, and only then is it given suitable semantics (e.g. Wolfgang Lenzen's (1980) epistemic systems); (3) as formal systems whose construction is started with ready-made semantics (which can be a fragment of physical

[14(] The considerations presented here were to a great extent prompted and influenced by the research conducted in the Department of Logic and Theory of Knowledge of the Catholic University of Lublin. My deep gratitude is to Prof. Ludwik Borkowski, who has supervised this research. In this part I shall use further (with little modifications) my analyses presented in Ch. II Interpreted modality (in Polish) of my book (Żegleń 1990).

or philosophical theory) and then a formal language for it is constructed (e.g. in the case of some temporal and deontic systems).

2.1 Epistemic modality

In this part of my paper I shall deal with epistemic modality. Both in their analyzes of epistemic notions, and in debates in the philosophy of logic, logicians address the problems raised by philosophers in epistemology. Thus there is a good reason for designing formal systems based on the notion of knowledge or belief, which is an allusion to the Platonic distinction between cognition in the sense of *episteme* (certain cognition) and *doxa* (opinion). Analytic studies emphasize the differences between these two types of cognition, pointing out that the cognition in the sense of knowledge, in contrast to belief, is certain and is not subject to the "true"/"false" qualification, while belief can be both true and false (Prichard 1973).[15] In addition, it is customary to distinguish various forms of epistemic utterances, e.g. "knows that" from "knows how" or "knows where/when" (Hintikka 1962, Carr 1979).

Philosophy of logic raises many questions concerning these two conceptions of cognition, namely: does cognition in the sense of knowledge entail cognition as belief, and if so, is it possible to define the former in terms of the latter, i.e. as a true belief? Some scholars also underscore, in connection to Plato, the differences between cognition that refers to a proposition (*episteme*) and cognition referring to things (*gnosis*).

The philosophical issues discussed in epistemic logic often suggest a radical idea that this type of logic should be regarded as a formalized theory of knowledge. This, however, seems too unreasonable a requirement for a formal system, which employs methods different from the philosophical ones. Some authors (e.g. Ilkka Niiniluoto 1979) consider the deductive method of epistemic logic mainly as a way to clarify and analyze philosophical problems. This is the analytic view of philosophical issues, quite different from a methodological perspective of classical philosophical approaches. Then, it is not logic which is to serve as a formal tool to elucidate philosophical problems (which however, to a certain extent, it is able to accomplish), but it is philosophy or, more precisely, ontology and – in the case of epistemic logic – philosophical epistemology that is supposed to illuminate formal analyzes of philosophical notions.

[15]The author quotes Descartes when he says that just as cognition in the sense of *knowledge* is neither true, nor false, so colours are neither heavy, nor light.

Philosophy also inspires further formal investigations and criteria for the assessment of more adequate accounts of philosophical concepts. Yet, here we should bring out all limitations and simplifications imposed on formal systems which aspire to formalize philosophical concepts. The main cause of these limitations are differences between the rich content of philosophical language and the much poorer formal languages. It seems, therefore, that it is the logician with philosophical aspirations that can avail herself of the rich philosophical arsenal, which contains richer methods and is aimed at the maximalist account of reality. It does not mean, however, that philosophers should not deploy formal tools (say, to achieve greater precision of their analyzes), or that logicians should doubt the usefulness of their investigations in the field of philosophy, which allow us to reveal ontological assumptions of deductive systems or philosophical implications of their theorems.

Therefore, the considerations I wish to present here will, to a certain degree, combine formal and philosophical goals. I shall give some examples of formulas from epistemic logic to show how the above-mentioned modalities (which are philosophical concepts) function in a formalized form in deductive systems, and what philosophical consequences that has. In this way I shall bring attention to formal analysis of those concepts, i.e. to the ontological assumptions that led to certain formal accounts of them, as well as to ontological and formal consequences of the accounts. The present fragment of the work will belong to philosophy of language, founded, however, on a maximalist understanding of philosophy (as a coherent system of philosophical disciplines founded on ontology which is strictly connected with epistemology).

I shall begin the discussion of formal issues with some aspects of epistemic logic.[16] It is constituted by deductive systems which formalize epistemic utterances. The most formally advanced among them are based on the concept of knowledge, i.e. they formalize utterances of the form: (1) "a knows that p", i.e. Kap. Some (e.g. Malcolm 1973) distinguish two meanings of the verb *to know*: the strong one – in the sense of having analytic knowledge (e.g. "a knows that $p \leftrightarrow p$"), and the weak one – in the sense of having knowledge of a weaker assertion than the analytic knowledge (e.g. "a knows that London is on Thames"). The formalization of the second type

[16]I shall appeal to some systems presented in publications which belong already to classical works in the contemporary literature in the field of logic and philosophy. G.H. v. Wright's book from 1951 is sometimes mentioned as that one in which first time epistemic logic has been presented. On the overview of epistemic logic in the recent literature see: Gochet, Gribomont (2006).

of cognition (i.e. in the sense of belief) was given by Hintikka (1962), who analyzes utterances of the form (2) "a believes that p", i.e. Bap. Systems based on this type of cognition are also called doxastic and they define expressions such as: (3) "It is possible, for all that a knows, that p", i.e. Pap (Hintikka), (4) "a considers it possible that p", i.e. Map, or "a is convinced that p", i.e. Uap (Lenzen 1970), and (6) "it is compatible with all a knows that p", i.e. Cap (Hintikka).

A separate group of epistemic utterances is formed by expressions in which the operator "knows that" is weakened to "know if / where, when" or "knows how", e.g. (7) "a knows if p", i.e. "a knows that p or a knows that not-p", which is formally rendered as K$ap \vee$ K$a\sim p$ (Hintikka), and (8) "a does not know if p", i.e. "a does not know that p and a does not know that not-p", i.e. $\sim$ K$ap \wedge \sim$ K$a\sim p$, as well as (9) "a knows how to do p" (Hintikka).

Sometimes (e.g. Hintikka 1975, Niiniluoto 1979), epistemic logic is enriched with perceptual expressions such as: (10) "a sees that p", i.e. Sap, (11) "a hears that p", i.e. Hap. Finally, one can analyze utterances concerning cognition of another person: (12) "a knows who b is", i.e. "a knows that $(b = x_1)$ or a knows that $(b = x_2)$, or ..., a knows that $(b = x_n)$", where x_1, x_2, ..., $x_n$are individual variables (Hintikka), (13) "a knows b", (14) "a remembers b" (Russell 1910).

Here I shall refer to a formal analysis of epistemic utterances falling within range of true cognition, i.e. knowledge. I will omit, however, the whole large debate on the problem of true cognition, which requires the consideration on truth, and given the satisfactory conditions for ascribing knowledge (as in the analysis of a knows that p).

Epistemic logic analyzes true cognition only with respect to its linguistic characteristics, by formalizing cognition *qua* product. Epistemic utterances falling within range of true cognition were formalized by Rescher (1974), who offered one of the most formally advanced type of epistemic logic. Rescher considers certain cognitive utterances consisting of true sentences about epistemic concepts. The aim of his epistemic logic, therefore, is not to characterize the actual cognition. Nevertheless, the epistemological problems implied by this type of logic and the attempt to solve them seem philosophically interesting. Let us start with presenting Rescher's systems. Their primitive term is the operator K, read as *knows that*.

A. Epistemic system corresponding to system T of normal modal logic; let us call this system T_e:

Axioms of T_e:

K1. $\mathrm{K}ap \rightarrow p$

K2. $\mathrm{K}a(p \rightarrow q) \rightarrow (\mathrm{K}ap \rightarrow \mathrm{K}aq)$

K3. $A \Rightarrow \mathrm{K}aA$

A is a theorem of logic, the sign "$\Rightarrow$ " can be understood as a metalogical operator of entailment; so K3 can also be laid down as a rule, which is known as the rule of epistemization (by analogy to the necessitation rule from normal modal logic).

RE. $\frac{\vdash A}{\vdash KaA}$

B. Epistemic system S_e4 (corresponding to system S4 of normal modal logic) contains axioms of system T_e and:

K4. $\mathrm{K}ap \rightarrow \mathrm{K}a\mathrm{K}ap$

C. Epistemic system Se5 (corresponding to system S4 of normal modal logic) contains axioms of system T_e and:

K5. $\sim \mathrm{K}ap \rightarrow \mathrm{K}a\sim \mathrm{K}ap$

First, let us try to bring out some problems implied by these axioms. The interpretation of K1 makes it clear that we are dealing with true knowledge, i.e. $\mathrm{K}ap \rightarrow p$: "if a knows that p, then it is the case that p" (in semantic formulation the consequent is read as: the state of affairs described by p obtains), so what is known by the subject is really the case. On the other hand, K3 shows the scope of one's knowledge – $A \Rightarrow \mathrm{K}aA$: "if A is a theorem of logic, then one knows that A", so it can be assumed that one knows every theorem of the system. K4 and K5 point to one's metaknowledge, i.e. awareness of one's own knowledge or ignorance – $\mathrm{K}ap \rightarrow \mathrm{K}a\mathrm{K}ap$: "if a knows that p, then a knows that a knows that p"; and $\sim \mathrm{K}ap \rightarrow \mathrm{K}a\sim \mathrm{K}ap$: "if it is not the case that a knows that p (which is read as: "a does not know that p"), then a knows that a does not now that p". K4, called the "KK principle" by logicians, is among the most discussed axioms (e.g. Hintikka 1962, Lehrer 1970, Feldman 1981).[17] It expresses the 'meta-' character of cognition, which – as emphasised in philosophical epistemology – is achieved thanks to reflexivity of cognition. Reflexivity consists in the fact that every cognition, and so every cognitive act and each of its products, can become an object of other cognitive act or of other product (result) of cognition.

By stressing the 'meta-' character of cognition, logic points out not only that $\mathrm{K}ap$ implies $\mathrm{K}a\mathrm{K}ap$, but also that they are equivalent, i.e. $\mathrm{K}ap \leftrightarrow \mathrm{K}a\mathrm{K}ap$. Thus, it is needed to consider the meaning of the KK principle. In

[17]The issue is deeply rooted in ancient and medieval epistemology: it was discussed by philosophers such as Plato in *Charmides*, Aristotle in *Nicomachean Ethics*, St. Augustine in *De trinitate*, and Thomas Aquinas in *Summa theologica* II and *Questiones de quolibet* III (cf. Hintikka 1962: 107).

Rescher's systems discussed here, it is clear that the principle merely has a logical meaning, since axiom $\mathrm{K}ap \rightarrow \mathrm{K}a\mathrm{K}ap$ was introduced as a result of epistemic interpretation of the analogous axiom from the alethic system S4, i.e. $\mathrm{L}p \rightarrow \mathrm{LL}p$. In philosophy of logic, the meaning of the KK principle is explicated by means of a precise linguistic analysis of the expression "a knows that a knows that p", whereas – based on what has been said here about philosophical epistemology – one might think, without undermining formal analyzes, that what "a knows that a knows that p" means is that one can self-reflectively talk about one's cognition thanks to the fact that cognition of a sentence (or a state of affairs) p has been grasped as an object of cognition (self-reference of cognition). Some (like Hintikka) draw attention to a link between the KK principle and axiom K5, by claiming that both formulas express metaknowledge. Axiom K5, however, is usually mentioned as an example of a paradox of epistemic logic, given that ignorance leads to knowledge about this ignorance. Lack of knowledge is characterized in yet another way by a rule put forward by Rescher (1974: 103) (sometimes called the "principle of ignorance"), i.e. PI. $\frac{\sim KaA}{KaB}$provided that $\vdash \sim \mathrm{K}aA \Rightarrow B$. Let us illustrate PI with an example: take the sentence "John does not know who is the president of Poland in 2014" as $\sim \mathrm{K}aA$. Can we conclude from this that John possesses other knowledge, e.g. $\mathrm{K}aB$, where $\mathrm{K}aB$ is the sentence "John knows that Tusk is the Prime Minister of Poland in 2014"? The example seems quite intuitive although in the condition $\sim \mathrm{K}aA \Rightarrow B$ there is no conceptual link between $\sim \mathrm{K}aA$ and B.[18] On the other hand, if we assume that $\sim \mathrm{K}aA$ is true, then $\mathrm{K}aB$ and B are also true.

Other paradoxical examples have also been quoted, resulting from intensionality of the language of epistemic logic. Horst Wessel (1984: 30–31), for instance, analyzes the following sentence: "Lila Miller knows that Stendhal wrote *The Red and the Black*". The sentence is true although "Lila Miler knows that Beyle wrote *The Red and the Black*" is false, despite the fact that Stendhal and Beyle are the same person, which Lila is unaware of. Thus p in the expression $\mathrm{K}ap$ cannot be substituted with q even though $p \leftrightarrow q$ (intensional languages violate the principle of extensionality). In Rescher's epistemic logic the intensional epistemic language has been stripped of any content, since the cognitive expressions have been specified merely extensionally. Cognition in the strict sense has been characterized in this way – its definition runs as follows: $\mathrm{K}aA \leftrightarrow \mathrm{K}^*aB \wedge \vdash_S B \Rightarrow A$", i.e. "$a$ knows that A if and only if a actually knows that B, and on the grounds of system

[18] Again I adress my thanks to Wojciech Wciórka for adding some remarks to this part of analysis.

S, *A* follows from *B*". What does it mean that one actually knows that *B*? Keeping in mind that knowledge discussed here is analytic in the framework of some deductive system, "*a* actually knows that *B*" means that *B* is an axiom, or that *B* is a theorem obtained in a deductive way, and subsequently *A* is deductively derived from *B*. Hence, "to know in the strict sense" means "to derive deductively".

Let us illustrate this with an example: let *B* be axiom A3 L$p \rightarrow p$ from system T, which will play the role of system *S* from the above definition. Then, in T one can derive the theorem $p \rightarrow$ Mp. According to the description given here, we assume that one actually knows axiom A3 L$p \rightarrow p$ and deductively derives from it the theorem $p \rightarrow Mp$ in T, which means that by knowing A3 in T, one also knows the theorem $p \rightarrow$ Mp. Apparently, the knower is treated here not as a person but as some perfect entity which – as axioms and theorems of Rescher's systems reveal – is omniscient.[19] This omniscience, however, is not absolute but relative. Under a stronger sense of the epistemic operator "knows that" we get systems in which the knower has been formalized also as a deductive system. Thus Rescher's epistemic logic analyzes true cognition (understood as a result), characteristic of deductive systems.

Let us now examine how epistemic logic analyzes utterances falling within range of the second type of cognition, i.e. cognition in the sense of belief (uncertain). As said before, cognition considered as belief encompasses both truth values, so we cannot accept that if a believes that *p*, i.e. Bap, then the state of affairs (described by) *p* obtains. Thus the formula analogous to K1 from system T is rejected here, i.e. (15) $\dashv$ B$ap \rightarrow p$. Following Hintikka, however, a formula analogous to K4, that is, (16) B$ap \rightarrow$ BaBap is adopted, i.e. we can speak of the BB principle here. It can be read as follows: "if one believes that a state of affairs (described by) *p* obtains, then one believes that one believes that the state of affairs *p* actually obtains"; the reverse implication is refuted. Next, attention has been drawn to the link between the expressions "to know" and "to believe". Formally, this connection is expressed by the formula (17) K$ap \rightarrow$ Bap, that is, "if *a* knows (it can be added: truly) that *p*, then *a* believes that *p*". It is also specified when Bap follows from Kap, namely, "if *a* believes that *p*, and *p* is actually the case, and *a* has adequate evidence that *p*, then *a* knows that *p*", which is

[19] In another way omniscience is interpreted in C.A. Meredith's system EM where there is the axiom according to which the knower knows the complete description of the world. Meredith belongs to first contemporary logicians who undertook the attempt at solving the problem of omniscience in logic (Meredith 1956).

written down as: (18) $\mathrm{B}ap \wedge p \wedge \mathrm{E}ap \rightarrow \mathrm{K}ap$ (Niiniluoto 1979: 254). (18) is a sort of answer to the issue, much discussed in philosophy of logic and in epistemology, of the relationship between the two types of cognition (and mainly it is the answer to the traditional question "when (some-)one comes to knowledge?". Yet the lack of formal specification of the operators with rich epistemic content indicates imperfection of formalization and is vulnerable to many philosophical objections. As mentioned before, the analysis of this sort of cognition requires a rich philosophical characterization, which is absent from the discussed formalizations.

Discussions centred around cognition in the sense of belief also concern certain important issues of philosophy of logic. One of the more notable among them is Moore's paradox and the problem of quantification into modal contexts, particularly the epistemic ones. The issue connected with Moore's paradox is summarised in the slogan "saying and disbelieving", which means that a state of affairs (described by) p obtains, but a does not believe in it, that is, (19) $p \wedge \sim \mathrm{B}ap$. Hintikka (1962: 64–76), for instance, analyzes formula (19), by considering the following situations:

(1) when a is referred to in first person, i.e. "p but I do not believe that p", that is, (20) $p \wedge \sim \mathrm{B}a_I p$ – such an utterance is paradoxical;

(2) when a is referred to in third person, as in (19);

(3) when someone gives an account of the situation, i.e. "b believes that the case is as follows: p and a does not believe that p", which is formally rendered as: (21) $\mathrm{B}b(p \wedge \sim \mathrm{B}ap)$. The paradox comes about only if $b = a$, that is, when (21) is about one and the same person.

The second problem concerns quantification into modal contexts. The discussion was initiated by Quine who impugned modal contexts due to their referential opacity. Let us quote a well known example from the logico-philosophical literature: (a) "a knows that the dictator of Portugal is Dr. Salazar". The sentence admits of two readings – the transparent one and the opaque one; according to the transparent interpretation, (a) is implied by: "a knows that the dictator of Portugal is b" together with "$b =$ Dr. Salazar", and then (a) can be quantified, resulting in (21) "$\exists x\, \mathrm{K}a$ (the dictator of Portugal is x)". By contrast, on the opaque reading, (a) cannot be inferred from "a knows that the dictator of Portugal is b" together with "$b =$ Dr. Salazar", and consequently we cannot quantify across the epistemic operator in (a). The objection against quantifying epistemic statements (and modal utterances in general) turns on the fact that they do not perform the referential function, at least according to the opaque reading. There are also other examples ("a believes that Pegasus exists", "George IV does not know that Walter

Scott is the author of *Waverley*") which reveal unwelcome consequences of such quantification. It is clear that these difficulties, already known from the classical logic, are brought about by the existential interpretation of the discussed quantifier and are entangled in the problem of nonexistent objects.

The possible worlds semantics admits of existence of such objects in some possible world (Kripke). Accordingly, for instance, semantics for epistemic logic can include worlds compatible with one's knowledge, in the sense that one can also possess a negative knowledge (ignorance). For example, if George IV did not know that Walter Scott was the author of Waverley, then a certain position concerning possible worlds admits of a world constituted by his ignorance (on the object-oriented construal, such a world may consist of possible states of affairs, i.e. those which he does not know to exist as well as those which he believes not to exist). It is already seen that in analyzes of this sort the knowing agent is regarded as a person. It is even clearer in formulas about two agents. Those formulas include (22) $\mathrm{K}a\mathrm{K}bp \rightarrow \mathrm{K}ap$, that is, "if a person a knows that other person b knows that p, then the person a knows that p". For instance, if John knows that his friend Peter knows that Rafał Blechacz won the Chopin Piano Competition, then it follows that also John knows it. The formula illustrates the transmissibility of the results of cognition. However, adequacy of this notation requires a more definite specification of the meaning of the implication occurring here, because otherwise the interpretation of (22) is richer than the content of (22) itself. Similar formulation is not acceptable in the case of belief, that is, (23) $\dashv \mathrm{B}a\mathrm{K}bp \rightarrow \mathrm{K}ap$, so it does not follow from "person a believes that other person b knows that p" that "a knows that p".

A separate group of epistemic utterances is formed by perceptual expressions occurring in the logic of perception. Also here one can propose a principle analogous to KK, i.e. the SS principle: (24) $\mathrm{S}ap \rightarrow \mathrm{S}a\mathrm{S}ap$, that is, "if a sees that the state of affairs (described by) p obtains, then a sees that a sees that p obtains". The act of perception is metacognitive in character here. Operator S is specified by the following axioms (Niiniluoto 1979: 252):

S1. $\mathrm{S}a(p \rightarrow q) \rightarrow (\mathrm{S}ap \rightarrow \mathrm{S}aq)$

S2. $\mathrm{S}a(p \leftrightarrow q) \rightarrow (\mathrm{S}ap \leftrightarrow \mathrm{S}aq)$

S3. $\mathrm{S}aT$, if T is a propositional tautology

S4. $\mathrm{S}a(p \wedge q) \rightarrow (\mathrm{S}ap \wedge \mathrm{S}aq)$

S5. $\mathrm{S}ap \rightarrow \mathrm{S}a(p \vee q)$

We should now inquire into the meaning of the formalized expression "to see". A formalization of the results of external perception is hardly acceptable here. Clearly, the presented axioms define relationships holding between

sentences, or one can say that they express a certain attitude of an agent towards the sentences. Then perhaps we can speak of some kind of intellectual perception here: if, for instance, one grasps an implication in the intellectual way, then one intellectually grasps the antecedent only if one intellectually grasps the consequent as well; and likewise for other operators. But if we want to contend that the operator S formalizes the content of some intellectual perception, then one should appeal to philosophical epistemology, which offers the adequate account of this sort of cognition, i.e. direct cognition. By analogy to K1, the logic of perception is sometimes designed (e.g. by Niiniluoto) as containing the theorem:

S6. S$ap \rightarrow p$,

that is, "if a sees that the state of affairs (described by) p obtains, then the state of affairs (described by) p actually obtains". This time, the expression "to see" is a name of an action (literally: of seeing). It can be said that if one perceives (sees) something, and it is assumed that one does not make a mistake, then it means that the perceived (seen) state of affairs (or object) occurs (exists). Hence the consequent of S6 is taken to be a result of external perception.

Still, some scholars (including Ayer and Hintikka) construe "to see" as an equivalent of "to appear", so "a sees that p" means : "it appears to a that a sees p" or "it seems to a that p". Then, for instance, one cannot infer from "a sees white Tatra Mountains peaks gleaming in the Sun" that white mountaintops are gleaming, because in fact those might be bare, matt rocks. Analysis of the examples illustrating the results of external perception reflects in a way the discussions in the field of philosophical epistemology between presentationists (such as phenomenologists: Edmund Husserl, Roman Ingarden or in quite different way nowadays analytic philosophers as John Searle for instance) and representationists (including such classical analytic philosophers, like Moore, Austin, Ayer, and Price) concerning the object of external perception. On the traditional representationist conception, S6 understood as earlier is controversial.[20]

I could quote further examples of formalization of the expression "to see" or other perceptual expressions, but due to a low degree of adequacy of these formalizations (from a philosophical point of view) it seems pointless. I had better choose one definite meaning of the expression (compatible with a philosophical account) and try to put forward – as far as possible – an adequate formal description of utterances of the Sap type. With an

[20]I omit the whole debate on the nature of perception in epistemology and newer approches in logic of perception.

epistemological account of perceptual utterances at hand, it would be possible to make out the differences between their various meanings. Yet this would be a semantic analysis, useful for elucidating the sense of perceptual utterances but utterly insufficient insofar as the nature of cognition itself is concerned. For in philosophical epistemology it is possible to distinguish many types of perception, and since it is a sort of direct cognition, an adequate account of it is possible only in the field of philosophical epistemology.

As I consider epistemological issues in their formal aspect, I should inquire into the semantics of epistemic systems and their problems. It is the possible worlds semantics. I has already mentioned, three accounts of possible worlds are the most popular ones: linguistic (started with Carnap), object-oriented (in different versions in David Lewis and Plantinga as main representatives), and epistemic (e.g. Hintikka, Rescher). The linguistic approach can be reconciled with the epistemic one because possible worlds are identified with a set of linguistic expressions which describe certain objects of intellectual processes. Thus sentences describing possible worlds are both about certain states of affairs occurring objectively (in ontological perspective) and about what is known or not known in those worlds (in epistemic perspective). I will leave aside formal characterization of world structures for particular systems of epistemic logic but I shall briefly discuss some of their interpretations.

First, let us try to use the possible worlds semantics to interpret some statements from the formal systems discussed here. Take axiom K1. Kap $\rightarrow$ p, "if a knows that p, then the state of affairs (described by) p obtains". It can be construed as saying that if an individual from the actual world knows that p, then in each possible world accessible from the actual world, the state of affairs (described by) p obtains. Since in the system T_e, from which this axiom is taken, the accessibility relation is reflexive, the state of affairs (described by) p occurs in the actual world. If we consider in turn systems in which the accessibility relation gains new properties, so that it is possible to access other worlds apart from one's own, then one knows that p only if the state of affairs (described by) p obtains in all those worlds accessible from the actual world w_i.

I used here the condition defining the values of alethic formula Lα, i.e. $V(\mathrm{L}\alpha,w_i) = 1 \leftrightarrow \forall w_i\ (w_iRw_j \rightarrow V(\alpha,w_j) = 1)$. In order to be able to interpret the theorems of epistemic systems, it is enough to replace operator L with K. It seems reasonable all the more because operator K expresses analytic knowledge, while analytic sentences are obtained in logic as true in every possible world (in Leibnizian spirit). Likewise, by analogy

to the systems of normal modal logic, we can interpret epistemic formulas in epistemic systems with accessibility relation regarded as transitive (S_e4 with axiom K4 suitable for illustrating this relation) and as symmetric (S_e5 with axiom K5 suitable, in turn, to illustrate symmetry).

Since, however, as follows from the above interpretations of Rescher's systems, the knowledge in question is analytic, and the knower is treated as a deductive system, it seems that the universe ventured in the possible worlds semantics will not be of much concern to a philosopher. Even if it is regarded most generally as encompassing the whole deductive knowledge, the discussion should still be directed towards philosophy of mathematics or philosophy of logic, and not towards the classical theory of being understood as the theory of reality. So if the crucial question faced by the possible worlds semantics is the question concerning the ontological status of possible worlds (mainly their mode of existence), then the answer should depend on the existence of objects in mathematics and logic. Besides, it seems that if Kripke's model is to be applied both to epistemic and normal modal logic, then it must, on account of the above analyzes, be modified in certain respects. These adjustments will above all depend on the ontological status of the realms of being to which the epistemic systems will refer. Accordingly, we propose the following modifications of Kripke's model:

I. $<C, C_i, R>$,

where $C = (C_1, ..., C_n)$ is a sequence of suitable sentential expressions and R – an inferential relation, which means that we are dealing with deductive systems (constituted by sets of linguistic expressions), which can stand to each other in relation of logical entailment. Thus it would be a model for Rescher's systems.

II. $<G, L, R>$,

where G signifies intentional states of affairs taking place in one's mind, L – a language, R – a relation of certain intentional correspondence between those intentional states of affairs and the language. We can assume, therefore, that actualization is realized here by verbalization, since it is the language which corresponds to the actual world from Kripke's model.

III. $<K, S, R>$,

where K is one's knowledge (true or not true), S – objectively obtained states of affairs, and R is also a relation of certain intentional correspondence between those states of affairs and the knowledge. This time actuality has been regarded as exemplification or concretization of knowledge in reality – as possible worlds are currently construed as one's knowledge. The concretization comes about in virtue of the classical criterion of truth (as

a certain correspondence between knowledge and its domain of objective states of affairs, as it is viewed from the realistic position).

It must be, however, taken into account that each of these models requires rich comments and evokes lots of controversial issues in different philosophical theories. Nevertheless it seems that these models cover the main layers of cognition (i.e. intentional states of affairs taking place in mind, their verbalized forms in language, and their concretizations in reality). Thanks to that, they can be assigned to different types of epistemic logic (i.e. model I to the systems which formalize the true cognition – knowledge, while two other models to the systems which formalize some aspects of cognition in the sense of belief).

2.2 Temporal logic[21]

The second type of modality that we are concerned of here, the temporal one, is among the oldest topics of logic and philosophy, as it was already considered by Aristotle and the Stoics. Temporal issues were vividly discussed by the Megarians (especially Diodorus Cronus), and in the Middle Ages – by Arabic philosophers and scholastics. Ancient and medieval analyzes of time are used today in constructing temporal systems. For instance, Diodorus's system has been reconstructed and shown to be identical with Lewis's system S4 or to be somewhere between S4 and S1.[22] Formal analyzes in contemporary logic are carried out in the following ways:

(1) as historical studies started by Arthur Prior (1955, 1958) (see also Prior 1967, Rescher 1974, part I among others),

(2) as analyzes of tense, especially of perfective and imperfective forms (D.M. Gabbay 1976, Jonathan F. v. Benthem 1977, 1984); they are frequently labelled "tense logic".

(3) as analyzes of time based on philosophy of science, especially philosophy of physics (Rescher 1969); they consist mainly in formalization of methods of inductive inference (Prior, Rescher). Logic of this sort is often called "logic of time".

It is not always possible to make a clear-cut division between systems of tense logic and the logic of time. It often turns out that an analysis of tense involves some model of physical time, as seen e.g. in Prior's systems, who

[21]Here I give my analysis (with little modifcation) from (Zegleń 1990), p. 116-125 .

[22]This was the work of Prior, who is considered the pioneer of temporal logic. Prior argued in favour of the first option by building a matrix for Diodorus's system which corresponds to system S4. However, Edward J. Lemmon has shown that this matrix is not adequate for S4, since it also makes true formulas which do not belong to S4 but to S4.2 and S4.3; cf. Prior 1957.

– starting with analysis of tense – achieved results enabling him to discuss physical and philosophical issues. For these metalogical reasons, I shall speak of temporal logic, without distinguishing between the two types. I will mainly use works by Prior (1957, 1967) and the philosophically interesting results achieved by Cocchiarella (1966), von Wright (1965), Lemmon 1977), and Rescher (1966, 1969).

Temporal logic consists of deductive systems formalizing certain temporal utterances (i.e. tensed sentences or simply statements built by means of a temporal operator). Temporal operators (treated here as temporal modalities) include:

(1) Fp – it will be the case that p,

(2) Pp – it has been the case that p,

(3) Gp – it will always be the case that p,

(4) Hp – it has always been the case that p.

The operators F, P, G, H, appear for instance in Prior's and Cocchiarella's systems.

Operators with indexes:

(5) $\mathrm{F}_n p$ – it will be the case n days hence that p,

(6) $\mathrm{P}_n p$ – it was the case n days ago that p.

Operators (1), (2), (5), (6) appear e.g. in Lemmon's systems.

Operators specifying temporal relations:

(7) Ypq – p now and q later,

(8) pTq – p and next q,

(9) $x < y$.– the instant x is earlier than the instant y,

(10) $x > y$ – x is later than y.

(7) and (8) appear e.g. in von Wright's systems, while (9) and (10) in Prior's.

By means of temporal modalities it is possible to define alethic modalities:

D1. M$p =_{df}\ p \vee \mathrm{F}p$

D2. L$p =_{df}\ p \wedge \mathrm{G}p$

D3. M$p =_{df}\ p \vee \mathrm{F}p \vee \mathrm{P}p$

D4. L$p =_{df}\ p \wedge \mathrm{G}p \wedge \mathrm{H}p$

Definitions D1 and D2 come from Diodorus Cronus: possible is what is or will be the case, and necessary is what is and will always be the case. These definitions are also adopted in contemporary systems: if D1 and D2 are added to Cocchiarella's temporal system, we get system $\mathrm{S}_t4.3$ (i.e. analogous to the alethic system), and if Cocchiarella's system is enriched with D3 and D4, we get in turn S_t5. Hence a suitable selection of axioms and definitions allows us to build temporal systems corresponding to Lewis's systems. D3

and D4 can be treated as accounts matching the Megarian conception of modality, according to which what is necessary is realized at each time. If one wishes to interpret these modalities in possible worlds semantics, then possible sentences would be true only in some possible worlds (actual, future, or past), while necessary sentences would be true in all possible worlds, provided that the worlds are temporally indexed.

From a philosophical point of view, the most interesting task would be to specify the ontological assumptions underlying temporal systems. The assumptions mostly concern the model of time intended for a formal characterization in a given system. The fundamental assumptions about time can be grouped as follows:

(1) regarding the structure of time: do assume that time is linear, branching, or circular?

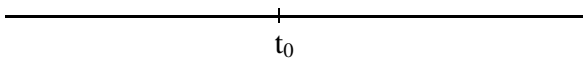
(2) regarding density: do we assume that time is continuous (i.e. between every two instants there is another instant, later than the former and earlier that the latter) or discrete (not continuous)?

(3) regarding determination: do we assume determinism?

In order to bring out the differences entailed by these assumptions, we will lay down the theorems implied by them. On the assumption that time is linear we get, e.g. in Cocchiarella's system the following theorems:

L1. $\mathrm{P}p \wedge \mathrm{P}q \rightarrow \mathrm{P}(p \wedge q) \vee \mathrm{P}(p \wedge \mathrm{P}q) \vee \mathrm{P}(q \wedge \mathrm{P}p)$

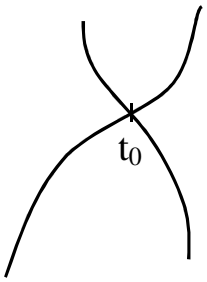
L2. $\mathrm{F}p \wedge \mathrm{F}q \rightarrow \mathrm{F}(p \wedge q) \vee \mathrm{F}(p \wedge \mathrm{F}q) \vee \mathrm{F}(q \wedge \mathrm{F}p)$



On the assumption that time is branching, in Cocchiarella's system we get, for instance, the following theorem:

B1. $\mathrm{P}(p \wedge \mathrm{F}q) \rightarrow \mathrm{P}(q \wedge \mathrm{P}p) \vee (q \wedge \mathrm{P}p) \vee (\mathrm{F}q \wedge \mathrm{P}p)$,

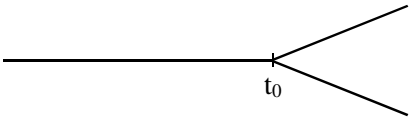
where the future and the past are branching:



or another theorem:

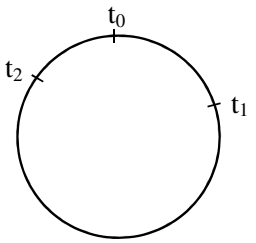
B2. $\mathrm{F}(p \wedge \mathrm{P}q) \rightarrow \mathrm{F}(q \wedge \mathrm{F}p) \vee (q \wedge \mathrm{F}p) \vee (\mathrm{P}q \wedge \mathrm{F}p)$,

where the future is branching:



Finally, under the assumption that time is circular, for example in Lemmon's system K_t, we get theorems such as:

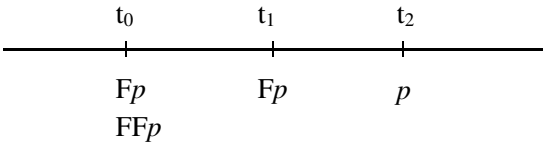
C1. $\mathrm{G}p \rightarrow \mathrm{H}p$
C2. $\mathrm{G}p \rightarrow \mathrm{PG}p$
C3. $\mathrm{G}p \rightarrow p$
C4. $\mathrm{F}p \rightarrow \mathrm{P}p$



Clearly, these theorems illustrate the so called 'mirror image' (particularly evident in C1 and C4). The mirror image rule states that expression α entails an expression β in which a past tense operator has been replaced with a future tense operator and vice versa, so P is replaced with F, H with G, and the other way round.

Let us now illustrate the assumptions connected with density. As a consequence of accepting the continuity of time, we get, e.g. in Hamblin's system, the following theorem:

D1. $\mathrm{FF}p \leftrightarrow \mathrm{F}p$



On the other hand, if time were discrete, given that $\mathrm{F}p$ is true at instant t_0 (cf. the diagram above), there would be no guarantee that there is an intermediary moment t_1 – later than t_0 and earlier than t_2– at which $\mathrm{F}p$ is true. Thus, in the case of discrete time, it would be impossible to infer $\mathrm{FF}p$

from Fp at t_0, which makes D1 false.[23]

Discrete time (with the last past moment) is illustrated by another theorem in Hamblin's system:

D2. GP$p \leftrightarrow\ p \vee$ Pp

Also the following Hamblin's theorems reflect the discreteness:

D3. $p \wedge$ G$p \rightarrow$ PGp (Diodorus formula)

D4. $p \rightarrow$ (G$p \rightarrow$ PGp)

D5. F$\sim p \wedge$ FG$p \rightarrow$ F($\sim p \wedge$ Gp)

D4 is a theorem that characterizes discreteness and enables us to obtain a logic of discrete time (e.g. as a result of adding D4 to system D of Dana Scott or Cocchiarella).

Yet another theorems characterize determinism or indeterminism. From a historical point of view, particularly interesting are reconstructions of Diodorus's and Aristotle's theses. For instance, we know the premises of Diodorus's famous Master Argument against indeterminists (chiefly against Aristotle): P$p \leftrightarrow\ \sim$ M$\sim p$ – "if it has been the case that p, then it is impossible that p should not be the case"; L($p \rightarrow\ q$) $\rightarrow$ ($\sim$ M$q \rightarrow\ \sim$ Mp) – "if it is necessary that if p then q, then it is impossible that q only if it is also impossible that p"; $\sim p \wedge \sim$ F$p \rightarrow\ \sim$ Mp – "if it is not the case that p and it will never be the case that p, then it is impossible that p".

We have presented the most fundamental ontological assumptions of temporal systems. Hence, apparently, certain temporal systems describe commensurate ontologies, with models of time set by their assumptions. This gives rise to a question whether certain physical theories, or at least their fragments, constitute semantics of certain temporal systems. But how does this look from the philosophical perspective? What ontology is described by temporal systems? Can it be found among known philosophical ontologies, or do we need to put it forward yet? Should we not make use of some semantic analyzes based on the concept of possible world? After all, temporal logic constructs systems of possible worlds and investigates formal relationships holding between those worlds (Prior). It is also possible to direct further research towards already known philosophical ontologies and to choose, for instance, Ingarden's ontology. Would the universe described in it, which individuates objects with respect to their relation to time, not be a model for some temporal systems? Clearly, like epistemic logic, temporal logic involves a lot of unsolved issues which invite further – philosophical rather than formal – consideration.

[23]I would like to thank Wojciech Wciórka for his correction of this piece of my text.

2.3 Deontic logic

The third kind of modality to be sketched here is the deontic modality. Deontic logic comprises deductive systems which formalize imperative utterances (ought-sentences). It serves as a formal device of deontology (also known as deontics), i.e. a general theory of obligation based on systems of norms. Although some scholars (e.g. Jerzy Kalinowski 1953) distinguish deontic logic (formal theory of all ought-sentences) from the logic of norms (formal theory of norms), they maintain that the former presupposes the latter, so they use both terms interchangeably. It seems reasonable that the name "deontic logic" has a broader extension and refers to all deductive systems which formally characterize the ought-sentences, whereas the logic of norms can be regarded as a subtype of deontic logic. In the case of formalizing legal norms we can speak of legal logic.[24] Nicholas Rescher, who is often quoted in research on non-classical logic, in his map of philosophical developments of logic in the field of ethical applications speaks about logic of action which includes deontic logic, logic of imperatives and logic of preferences and choices, i.e. games and decisions (Rescher 1968).

Deontic systems are constructed by means of deductive method. Axioms are selected from among ought-sentences taken from ethics, law, or legal science; alternatively, the choice of axioms can be purely intuitive. At the stage of constructing the system, the central focus is on its formal soundness, and only after the system has been completed the accent is shifted to the material consequences of such and such choice of axioms. At this stage, theorems derived from a given systems are examined as to their correspondence with ethical principles or legal norms. The crucial question is: how "rich" (in relation to ethics, law, or some legal theory) is the created system? On the formal side, deontic systems are treated as deductive theories of ought-operators. The following operators are considered fundamental (and at the same they constitute the elementary deontic modalities):

– the operator of obligation – O (corresponding to alethic necessity),

– the operator of prohibition – F (corresponding to alethic impossibility),

– the operator of permission – P (corresponding to alethic possibility).

In addition, in systems linked to decision theory, there is an operator of free choice (decision) – I (corresponding to alethic contingency).

Axiomatizations of deontic systems are usually based on the primitive term "ought to", that is, the operator of obligation – O.

[24] In Poland formal analyses of legal reasonings (started by Jerzy Kalinowski) were made by Edward Nieznański, Jan Woleński. Zygmunt Ziembiński among others.

Other deontic operators are introduced to the systems by means of the following definitions:

Definition of permission:

DP. $\mathrm{P}p =_{df} \sim \mathrm{O}\sim p$

Definition of prohibition:

DF. $\mathrm{F}p =_{df} \mathrm{O}\sim p$

Definition of free choice (optionality):

DI. $\mathrm{I}p =_{df} \mathrm{P}p \wedge \mathrm{P}\sim p$

The issue of definability of deontic terms raises the question of whether they are to be defined merely internally, i.e. only by means of deontic terms, or externally, i.e. by means of other modal terms.

It is also worth noting that in some systems the deontic operators are interpreted either objectually or meta-objectually, either statically or dynamically. Deontic operators interpreted objectually make up ought-sentences which perform the prescriptive function (of a command or prohibition):

$\mathrm{O}p$ – it ought to be the case (it is obligatory) that p,

$\mathrm{F}p$ – it is prohibited (forbidden) that p,

$\mathrm{P}p$ – it is permitted that p,

Ip – it is optional that p.

Apparently, a sentence (more precisely: a sentential variable) p refers to a state of affairs. In order to emphasize the objectual character of a statement, it is customary to use a different notation, i.e. to introduce symbol A which refers to an action that is obligatory, forbidden, permitted, or optional, symbol m which refers to a person which is supposed to perform this action, and a situational variable S; in this way we get expressions of the form: "m ought to perform action A in situation S", "Person m is permitted to do A in situation S", and the like.

Alternatively, deontic statements can be interpreted meta-objectually, when a sentence (more precisely: a sentential variable) p is understood as a norm (more precisely: a sentential variable representing a norm). On this construal, also the operators are read meta-objectually:

$\mathrm{O}p$ – p is an obligation (order),

$\mathrm{F}p$ – p is a prohibition,

$\mathrm{P}p$ – p is a permission,

Ip – p is an option (a free choice).

There is a problem with a proper understanding of arguments of such deontic operators, namely, what kind of sentences they are: assertoric sentences, like in classical logic, or ought-sentences. And if they are ought-sentences performing a prescriptive function, then could they be translated into as-

sertoric sentences playing a descriptive role? Another question is whether the ought-sentences, including a great deal of norms (ethical and legal), are logical in nature, that is to say, whether they can receive one of two truth values. Finally, are we allowed to speak of a normative inference, set its rules, provide the schemes? Is it a normal deductive inference with the sentences discussed here (but which: assertoric or normative?) serving as its premises?

The answers to those questions are not straightforward, they must be sought in methodology of legal or deductive sciences, in logical semantics, philosophy of language, and philosophical axiology. However, the answers given there differ from each other, depending on the philosophical and methodological presumptions. The most elementary among those are ontological assumptions concerning the nature of norms, which can be considered either in the realm of actual being (when a norm is ontologically dependent on a subject, as in the classical theory of being), in the realm of ideal being (when a norm exists objectively, independently from a subject – Platonism, contemporary phenomenology), in the realm of language (like in linguistic theories of norms, based on various kinds of philosophy of language), or in the realm of action (when a norm consists in rules defining agent's behaviour in society – nominalist theory of norms, founded on various types of praxist philosophy).

Ontological specification of the nature of norms determines, in turn, the methodological character of sentences counted among practical sentences. Deontic logicians, who formalize certain sorts of practical sentences, usually adopt a nominalist approach by treating norms as linguistic expressions with truth values. Yet, in discussions held in the framework of methodology of legal sciences, a position is admissible that strips norms of truth values due to the lack of analogy between validity of norms and the value of truth attributed to sentences in classical logic (Kelsen 1974). There are also dichotomies of positions regarding the possibility of translating ought-sentences into equivalent assertoric sentences (Schreiber 1977 – Weinberger 1977). The position admitting of the translatability also advocates the logical character of ought-sentences. Furthermore, it is sometimes claimed (Inhetveen 1977, Keuth 1974) that deontic logic formulates assertoric sentences about descriptive utterances, i.e. assertoric parts of prescriptive sentences. The problem of transition from assertoric sentences to ought-sentences (from the *is* operator to the *ought* operator) has also been among the crucial issues discussed in the analytic tradition since David Hume up today (Black 1964, Searle 1964).

Logicians who opt for prescriptive (not descriptive) nature of ought-

sentences in deontic logic, frequently write them down with an exclamation mark in order to emphasize that the arguments of deontic operators are imperative sentences, e.g. $!p$ as the argument of the expression with the operator $O!p$. As for the normative inference, one could employ the Aristotelian syllogistic of practical sentences, which was the first attempt to formally analyze such reasoning. Besides, rules can be regarded as imperative sentences performing a prescriptive function in a metasystem.

Thus deontic logic provides schemes of normative inference, by means of studying formal relationships holding between ought-sentences; this fact would be more evident, however, if the systems were created by using the assumption-based method, instead of the axiomatic one.

It is worth mentioning a formally interesting attempt to treat deontic logic dynamically, i.e. as a logic of change or action. Such an interpretation was put forward by von Wright (1963), who introduced to system's vocabulary the operator of transformation T. Formulas containing this operator (T-expressions) are read as follows:

$OpTq$ – it is obligatory to go from behaviour p to behaviour q,

$PFpT\sim Fp$ – it is permitted to stop the prohibition p,

$OOpTOq$ – it is obligatory to go from the order that p to the order that q.

It is easily seen that the introduction of the transformation operator admits of a dynamic reading of its arguments, which describe behaviour or action.

The above discussion has made it clear that deontic logic takes into account three aspects: syntactic – in formulating ought-sentences, semantic – in metalinguistic interpretation of these sentences, pragmatic – in emphasizing the role of action.

Concluding remarks

In conclusion it is worth to stress that modal logic with regards to its ontological commitment deserves special attention among the systems of non-classical logic. The paper focused not only on the alethic modalities, but first of all on interpreted modalities in epistemic, temporal and deontic logic. As has been showed these three types of modality are analyzed on three levels: syntactic, semantic and pragmatic. By their very nature, the discussed interpreted modalities are pragmatic in character, although this assessment is controversial in the case of temporal modality: is it not semantic in nature (since it examines relationships between sentences involving time)? On the other hand, however, analysis of temporal modality leads to

considerations about the realm of real being, and since they take existence into account, it is reasonable to point to the pragmatic character of this modality. Contemporary logic, therefore, formalizes pragmatic concepts, which play a role of suitable modal operators in a formal language, usually characterized axiomatically.

The modal concepts are then characterized semantically by means of Kripke's model. The model, generally speaking, is subsequently modified according to the needs of various systems. The following approaches are shared here: (1) the linguistic one, in which a possible world is a set of epistemic, temporal, or deontic expressions, (2) the object-oriented one, in which a possible world is a set of individuals existing in a possible way, (3) the epistemic one, according to which a possible world is a set of certain cognitive situations or of objects of intellectual processes. Each of the discussed types of logic implies distinct ontologies filled with different individuals; namely, the ontology of epistemic logic contains objects from the intentional realm (i.e. objects or states of affairs known or believed to obtain), the ontology of temporal logic contains time-related objects, belonging to the realm of real being; finally, the ontology of deontic logic contains items from the realm of obligation, whose description depends on the adopted philosophical position (e.g. in phenomenology it is ideal, in the classical theory of being – real but not independent).

Thus metalogical remarks made here point to rich ontological implications of non-classical logics, concerning the fields of study considered to be the domain of philosophical cognition. This is why contemporary logic is a philosophically intriguing subject, enabling one to get over purely formal accounts and reveal new ontological perspectives.

Bibliography

1. Adams, Robert M. (1979) *Theories of Actuality*. In Loux 1979: 190–209.

2. Anderson, Alan R. (1956) *The Formal Analysis of Normative Systems*. New Haven (CT): Office of Naval Research.

3. Armstrong, David M. (1989) *A Combinatorial Theory of Possibility*, Cambridge: Cambridge University Press.

4. Ayer, Alfred J. (1956), *The Problem of Knowledge*. London: Macmillan.

5. Becker, Oscar (1930), "Zur Logik der Modalität". *Jahrbuch für Philosophie und Phänomenogische Forschung* 11 (1930): 497-548.

6. Benthem, van Jonathan F. (1977) "Tense Logic and Standard Logic". *Logique et Analyse* 80 (1977): 395-437.

7. Benthem van Jonathan F. (1983) *The Logic of Time. A Model Theoretic Investigation into the Varieties of Temporal Discourse*. Dordrecht: D. Reidel (2^{nd} ed. 1991).

8. Benthem van Jonathan F. (1984) "Tense Logic and Time", *Notre Dame Journal of Formal Logic* 25 (1984) no 1: 1-17.

9. Black, Max (1949) Language and Philosophy. Ithaca (NY): Cornell University Press.

10. Black, Max (1964) "The Gap Between ŕIső and ŕShouldő". *The Philosophical Review* 73, no. 2: 165–181.

11. Blumenfeld, David (1972). "Leibniz's Modal Proof of the Possibility of God". *Studia Leibnitziana* 4, no. 2 (1972): 132–140.

12. Blumenfeld, David (1975) "Is the Best Possible World Possible?". *The Philosophical Review* 84, no. 2 (1975): 163–177.

13. Burkhardt, Hans (1980) *Logik und Semiotik in der Philosophie von Leibniz*. München: Philosophia.

14. Carnap, Rudolf (1947) *Meaning and Necessity*. Chicago (IL): Chicago University Press.

15. Carr, David (1979) "The Logic of Knowing How and Ability". *Mind* 88, no. 1: 394–409.

16. Castañeda, Hector Neri (1974) "Thinking and the Structure of the World". *Philosophia* 4, no. 1: 3–40.

17. Chisholm, Roderick M. (1967) "Identity Through Possible Worlds: Some Questions". *Noûs* 1, no. 1: 1–8.

18. Chisholm, Roderick M. (1973) "Beyond being and nonbeing". *Philosophical Studies* 24, no. 4: 245–257.

19. Chisholm, Roderick M. (1977) *Theory of Knowledge*. Englewoods Cliffs (NJ): Prentice-Hall.

20. Chisholm, Roderick M. (1979) "Events, Propositions and States of Affairs". In *Ontologie und Logik*, P. Weingartner, E. Morscher (eds.), 27–47. Berlin: Dunckner & Humblot.

21. Church, Alonso (1958) "Ontological Commitment". *The Journal of Philosophy* 55, no. 23: 1008–1014.

22. Cocchiarella, Nino (1966) "Modality within Tense Logic". *The Journal of Symbolic Logic* 31: 690–691.

23. Cocchiarella, Nino (1982). "Meinong Reconstructed versus Early Russell Reconstructed". *Journal of Philosophical Logic* 11, no. 2: 183–214.

24. Conte, Amadeo G., and Risto Hilpinen, G. H. von Wright (1977) *Deontische Logik und Semantik*. Wiesbaden: Athenaion.

25. Cresswell, Maxwell J. (1972) "The World Is Everything That Is the Case". *Australasian Journal of Philosophy* 50, no. 1: 1–13.

26. Feldman, Richard (1981) "Fallibilism and Knowing that One Knows". *The Philosophical Review* 90, no. 2: 266–282.

27. Fine, Kit (1975), "Critical Notice of *Counterfactuals*". *Mind* 84 (1975): 451-458.

28. Gabbay Dov M., Woods, John (eds.) (2006) *Handbook of the History of Logic*. Vol. 7. *Logic and Modalities in the Twentieth Century*, Amsterdam: Elsevier North Holland.

29. Goodman, Nelson (1977) *The Structure of Appearance*. Dordrecht: D. Reidel.

30. Gochet, Paul, Gribomon, Pascal (2006) "Epistemic Logic" In Gabbay and Woods, 2006: 99-196.

31. Griffiths, Allen Ph. (ed.) (1973) *Knowledge and Belief*. Oxford: Oxford University Press.

32. Hacking, Ian (1975) "All Kinds of Possibility". *The Philosophical Review* 84, no. 3: 321–337.

33. Hare, Richard M. (1952) *The Language of Morals*. Oxford: Clarendon Press.

34. Hintikka, Jaakko (1962) *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Ithaca (NY): Cornell University Press.

35. Hintikka, Jaakko (1963) "On the Logic of Perception". In Hintikka 1969: 151–183.

36. Hintikka, Jaakko (1969) *Models for Modalities: Selected Papers*. Dordrecht: D. Reidel

37. Hintikka, Jaakko (1974) *Knowledge and the Known*. Dordrecht: D. Reidel.

38. Hughes, Georg Edward, Cresswell, M. J (1972) *An Introduction to Modal Logic*, London: Methuen & Co Ltd

39. Hughes, Georg Edward, Cresswell, M. J. (1996) *A New Introduction to Modal Logic*, London: Routledge (completely re-written)

40. Inhetveen, Rüdiger (1977) "Die konstruktive Interpretation der modallogischen Semantik". In Conte et al. 1977: 89–100.

41. Kalinowski Jerzy (1953) "Teoria zdań normatywnych" ["Theory of Normative Sentences"]. *Studia Logica* 1: 113–184.

42. Kalinowski Jerzy (1953) *La logique de norms*. Paris: Presses Universitaires de France.

43. Kelsen, Hans (1974) *Essays in Legal and Moral Philosophy*. Dordrecht: D. Redel.

44. Keuth, Herbert (1974) "Deontische Logik und Logik der Normen". In Lenk 1974: 64–68.

45. Kripke, Saul (1963a) "Semantical Considerations on Modal Logic". *Acta Philosophica Fennica* 16: 83–94.

46. Kripke, Saul (1963b) "Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi". *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 9: 67–96.

47. Kripke, Saul (1972) "Naming and Necessity". In: *Semantics of Natural Language*, D. Davidson, G. Harman (eds.), 253–355, 763–769. Dordrecht: D. Reidel

48. Latinov, Eugine (2014) *Modal Logic of Strict Necessity and Possibility*, Open Access [https://www.otexts.org/mlo/1/13]

49. Lehrer, Keith (1970) "Believing that One Knows". *Synthese* 21, no. 2: 133-140. Lenzen, Wolfgang (1980) *Glauben, Wissen und Wahrscheinlichkeit: Systeme der epistemischen Logik*. Wien: Springer.

50. Lemmon, Edward J. (1977) [ed. K. Segerberg] *An Introduction to Modal Logic*. Oxford: Blackwell.

51. Lemmon, Edward J. and Dana Scott (1966) *Intensional Logic*. Stanford University.

52. Lenk, Hans (ed.) (1974) *Normenlogik: Grundprobleme der deontischen Logik*. Pullach: Dokumentation.

53. Lewis, David (1968) "Counterpart Theory and Quantified Modal Logic". *The Journal of Philosophy* 65, no. 5: 113–126.

54. Lewis, David (1971) "Counterparts of Persons and Their Bodies". *The Journal of Philosophy* 68, no. 7: 203–211.

55. Lewis, David (1978) "Truth in Fiction". *American Philosophical Quarterly* 15: 37–46.

56. Lewis, David (1979) "Possible Worlds". In Loux 1979: 182–189.

57. Lewis, David (1986), *On the Plurality of Worlds*, Oxford: Blackwell [new edition 2001].

58. Loux, Michael J. (ed.) (1979) *The Possible and the Actual: Readings in the Metaphysics of Modality*. Ithaca (NY): Cornell University Press.

59. Malcolm, Norman (1973) *Knowledge and Belief*. In Griffiths 1973: 62–82.

60. Meredith, C. M. (1956) *Interpretations of Different Modal Logics in the "Property Calculus"*. Christchurch: University of Canterbury.

61. Munitz Milton K (ed.) (1973) *Logic and Ontology*. New York (NY): New York University Press.

62. Munitz Milton K (1974) *Existence and Logic*. New York (NY): New York University Press.

63. Niiniluoto, Ilkka (1979) *Knowing that One Sees*. In Saarinen et al. 1979: 249–282.

64. Parsons, Terence (1974) "A Prolegomenon to Meinongian Semantics". *The Journal of Philosophy* 71, no. 16: 561–580.

65. Parsons, Terence (1978) "Nuclear and Extranuclear Properties, Meinong, and Leibniz". *Nous* 12, no. 2: 137–151.

66. Parsons, Terence (1980) *Nonexistent Objects*. New Haven (CT): Yale University Press.

67. Pelc, Jerzy (1983) "O bytach fikcyjnych i tekstach fikcjonalnych". *Studia Semiotyczne* 13: 179–224. English version: "On Fictitious Entities and Fictional Texts". *Recherches Sémiotiques – Semiotic Inquiry* 6, no. 1 (1986): 1–35

68. Plantinga, Alvin (1970) "World and Essence". *The Philosophical Review* 79, no. 4: 461–492.

69. Plantinga, Alvin (1973) "Which Worlds Could God Have Created". *The Journal of Philosophy* 70, no. 17: 529–552.

70. Plantinga, Alvin (1974) *The Nature of Necessity*. Oxford: Clarendon Press

71. Plantinga, Alvin (1976) "Actualism and Possible Worlds". *Theoria* 42: 139–160.

72. Prichard, Harold A. (1973) *Knowing and Believing*. In Griffiths 1973: 60–69.

73. Prior, Arthur N. (1955) "Diodorian Modalities". *The Philosophical Quarterly* 5: 202-213.

74. Prior, Arthur N. (1957) *Time and Modality*. Oxford: Oxford University Press.

75. Prior, Arthur, N. (1958) "Diodorus and Modal Logic". A Correction. *The Philosophical Quarterly* 8: 226-230.

76. Prior, Arthur N. (1967) *Past, Present and Future*. Oxford: Oxford University Press.

77. Prior, Arthur N. (1968) *Papers on Time and Tense*. London: Clarendon Press.

78. Przełęcki, Marian (1974) "O świecie rzeczywistym i światach możliwych" ["On the Actual World and the Possible Worlds"]. *Studia Filozoficzne* 7: 47–56

79. Quine, Willard V. (1948) "On What There Is". *The Review of Metaphysics*: 21–38.

80. Quine, Willard V. (1951) "On Carnap's Views on Ontology". *Philosophical Studies* 2: 65–72.

81. Quine, Willard V. (1968) "Propositional Objects". *Critica* 2, no. 5: 3–29.

82. Rescher, Nicholas (1966) "On the Logic of Chronological Propositions". *Mind* 75, no. 297: 75–96.

83. Rescher, Nicholas (1968), *Topics in Philosophical Logic*, Dordrecht: D. Reidel.

84. Rescher, Nicholas (1969) "Truth and Necessity in Temporal Perspective". In *Essays in Philosophical Analysis*. Univ. of Pittsburgh Press: 271–302.

85. Rescher, Nicholas (1974) *Studies in Modality*. Oxford: Blackwell.

86. Rescher, Nicholas (1975) *A Theory of Possibility*. Oxford: Blackwell.

87. Russell, Bertrand (1910) "Knowledge by Acquaintance and Knowledge by Description". *Proceedings of the Aristotelian Society* 11: 108–128.

88. Saarinen, Esa, Ilkka Niiniluoto, Risto Hilpinen, and Merrill P. Hintikka (eds.) (1979) *Essays in Honour of Jaakko Hintikka*. Dordrecht: D. Reidel.

89. Schreiber (1977) "Deontische Logik und Geltungstheorie". In Conte et al. 1977: 167–176.

90. Searle, John R. (1964) "How to Derive ŕOughtő from ŕIső". *The Philosophical Review* 73, no. 1: 43–58".

91. Sobociński, Bolesław (1953), "Note on a Modal Systems of Feys-von-Wright*", The Journal of Computing Systems* 1

92. Stalnaker, Robert C. (1979) "Possible Worlds". In Loux 1979: 225–234.

93. Von Wright, Georg H. (1951a) "Deontic Logic". *Mind* 60, no. 237: 1–15.

94. Von Wright, Georg H. (1951), An Essay in Modal Logic, Amsterdam: North Holland.

95. Von Wright, Georg H. (1963) *Norm and Action: A Logical Enquiry*. London: Routledge and Kegan Paul.

96. Von Wright, Georg H. (1965) "And Next". *Acta Philosophica Fennica* 18: 293–304.

97. Von Wright, Georg H. (1968) *An Essay in Deontic Logic and the General Theory of Action*. Amsterdam: North Holland.

98. Weinberger (1977) "Normenlogik und logische Bereiche". In Conte et al. 1977: 177–212.

99. Wessel, Horst (1984), *Logik*. Berlin: Deutscher Verlag der Wissenschaften.

100. Ziemińska, Renata (1998), *Epistemologia Rodericka M. Chisholma*, Bydgoszcz: Wydawnictwo Uczelniane Wyższej Szkoły Pedagogicznej.

101. Żegleń Urszula M. (1990), *Modalność w logice i w filozofii. Podstawy ontyczne* [Modality in Logic and in Philosophy. Ontic Foundations], Warszawa: Zakład Semiotyki Logicznej UW, Polskie Towarzystwo Semiotyczne.