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ON THE CONCEPT OF A LINGUISTIC RELATION

The article will focus on the concept of a linguistic relation. It will discuss characteristic features of this concept, as well as the classification of linguistic relations. In the latter section, it will consider the feasibility of a comprehensive presentation of a language system as a set of units and linguistic relations.

1. Linguistic relations will be treated extensionally, i.e. as defined by objects between which they occur. In other words, linguistic relations will be analysed as subsets of Cartesian products of the appropriate sets of objects, that is as sets of pairs, triplets, quadruples, etc. of linguistic objects. Thus, the concept of a linguistic relation is closely linked to the concept of a linguistic unit, and so it is the latter that needs to be discussed at this point.

Linguistic units are determined (introduced) with respect to various criteria. Accordingly, we are dealing with various types of linguistic units. Linguistic units are defined with respect to various aspects of the structure of language. Hence there are units of the expression plane (e.g. sounds, phones, phonemes, morphs, words, spoken words, written words, phrases, sentences, texts), units of the content plane (e.g. concepts, judgements, meanings, elements of meanings, extensions, intensions, senses), concrete units (i.e. tokens), such as e.g. sounds, morphs, utterances, and abstract units (i.e. types), such as phones, phonemes, syntactic categories, morphemes, etc.

Abstract linguistic units may be constructed by means of the resources of formal logic, as for instance, sets of concrete units. This is the case of

e.g. phones treated as sets of homophonic sounds; similarly, phonemes may be defined as specific sets of phones. Also, abstract units may be treated as indefinable or derived from other sciences (e.g. concepts, some acoustic units, etc.).

By concrete linguistic units we understand primarily segments of the utterance, that is spatio-temporally distinct sections of speech, that is sounds, syllables, morphs, concrete words, phrases, sentences, texts, etc. It must be emphasized that concrete linguistic units, arrived at by segmentation of the utterance, are generally not given directly — already their definition assumes the existence of some theoretical apparatus, both strictly linguistic and derived from other sciences. This is clearly seen in the case of utterances (1)—(4), cited below after (Chomsky and Miller 1963):

(1) *The good candy came anyway.*
(2) *The good can decay many ways.*
(3) *Gal, amant de la Reine alla (tour magnanime).*
(4) *Galamment de l'arene á la Tour Magne, à Nîmes.*

Linguistic units are interconnected by various relationships; thus, we speak of the homophony of sounds, the synonymy of words, agreement between clauses of a sentence, etc. as relations (connections) between relevant units. It must be remembered, however, that neither are the linguistic units primary with respect to the relevant linguistic relations, nor the other way round (at least in the case of concrete linguistic units and relations, that is ones arrived at by the segmentation of an utterance). There exists a mutual connection between them: singling out linguistic units presupposes the existence of definite relations between them and vice versa: linguistic relations occur always between definite units. This relationship has been pointed out by Saussure and Hjelmslev; its existence is assumed by contemporary theories of language as well.

Extensional treatment of linguistic relations follows, in a sense, the principle of not multiplying entities beyond necessity: instead of using two indefinable terms, linguistic unit and linguistic relation, we treat linguistic relations as set-theoretical constructs defined by linguistic units. A concrete example is the simplest method of explaining the issue. Let us consider a set of sounds (in a given language) and the relation of homophony between sounds. This relation occurs between distinct sounds, and thus it always connects two objects within the universe of sounds under consideration. This relation is, therefore, wholly and unequivocally defined by giving a catalogue of all pairs of sounds between which it occurs. Thus, the relation of homophony may be identified with the set of all pairs of homophonous

sounds. This set is a certain subset of the binary Cartesian product of the set of sounds, i.e. the set of all ordered pairs of sounds. Thus, in the end, the relation of homophony is a certain subset of the binary Cartesian product of the set of sounds.

Two-argument relations, that is relations between two linguistic objects (like the relation of homophony discussed above), are the most often occurring linguistic relations. Apart from them, in the study of language we also encounter relationships that link more than two objects (e.g. the three-argument relation of the degree of synonymy: word X is closer in meaning to word Y than to word Z). Also the properties (features) of linguistic objects may be examined as relations of a kind, i.e. one-argument relations. This involves an extensional treatment of properties: a given property is identified with the set of all objects that possess this property (e.g. the property of being a noun is identified with the set of all nouns). Subsets of a given universe, which are delineated by given properties, are known as one-argument relations in that universe.

2. Linguistic relations can be grouped into definite types both with respect to their general properties and with respect to the type of objects between which they occur. A large group of linguistic relations are various SIMILARITIES and EQUIVALENCES, i.e. relations which rely on non-distinguishability of units with respect to the selected set of features, e.g. the relation of homophony. Another group of linguistic relations are ORDERINGS of various sorts, from the simplest, like the linear ordering of the segments of utterance in time, to the more complex, like the hierarchic ordering of words with respect to hyponymy or the tree ordering of syntactic units in sentences. An important role in the structure of language is played also by the comparison of units with respect to their distribution in appropriate environments (contexts), for instance homodistribution (occurrence in exactly the same contexts) or complementary distribution (absence of common contexts). Relations based on distribution combined with relations based on meaning give various types of OPPOSITIONS as a result.

The division of linguistic relations into syntagmatic and paradigmatic is very widespread in linguistics. Yet these terms, albeit very frequently used, are often imprecisely described. Let us, therefore, focus on a relatively accurate definition of the syntagmatic and paradigmatic relations.

SYNTAGMATIC relations link linguistic units into larger segments, hierarchically higher than those units. In other words, linguistic units linked by a syntagmatic relation create a hierarchically higher unit, of which they are components. All linear relations, based on the succession of units in

the speech flow, the relations of parataxis and hypotaxis, agreement, text cohesion, etc. are examples of syntagmatic relations.

With respect to the types of units between which they occur, syntagmatic relations can be divided into CONCRETE and ABSTRACT ones. Syntagmatic relations that link concrete linguistic units (that is segments of an utterance) into hierarchically higher concrete units are concrete syntagmatic relations; other syntagmatic relations are abstract relations. The succession of sounds in the speech process, which binds those sounds into larger segments, is an example of a concrete syntagmatic relation. The relations of parataxis and hypotaxis, which bind words into phrases, may serve as examples of abstract syntagmatic relations.

Concrete syntagmatic relations provide a basis for the stratification of the universe of all the concrete linguistic units into STRATA. Each linguistic stratum is delineated by a group of syntagmatic relations that unite concrete linguistic units of one type into hierarchically higher units. In other words, units of a linguistic stratum are segments composed of units of a hierarchically lower stratum, linked by concrete syntagmatic relations. Essentially, hierarchic segmentation of an utterance into concrete linguistic units is based on the consideration of concrete syntagmatic relations between those units. The universe of concrete linguistic units is thus split into the strata of sounds, concrete syllables, morphs, concrete spoken words, concrete written words, concrete phrases, etc. It needs to be noted that concrete syntagmatic relations always occur between units belonging to the same linguistic stratum. Some additional observations on the mathematic model of the universe of concrete linguistic units divided into strata by syntagmatic relations can be found in (Pogonowski 1981: ch. 7).

Isolating those linguistic strata gives a foundation for defining PARADIG-MATIC relations. Similarly to syntagmatic relations, paradigmatic relations are also divided into CONCRETE and ABSTRACT ones. Concrete paradigmatic relations are those relations between units belonging to one linguistic stratum which are not syntagmatic relations. Concrete paradigmatic relations, in turn, may be divided into two types. The first type encompasses those relations which make it possible to form abstract units from concrete units (e.g. the relation of homophony between sounds, whose equivalence classes constitute abstract units, i.e. phones). The second type encompasses other paradigmatic relations, for instance various oppositions or the facultative variation of sounds.

Concrete paradigmatic relations of the first type make it possible to construct ABSTRACT LINGUISTIC STRATA, that is sets of abstract units

rendered by concrete units belonging to one linguistic stratum. Abstract linguistic strata are, for example, the phone stratum, the morpheme stratum, the word stratum, the sentence stratum, etc.

Abstract units belonging to one linguistic stratum also stand in (abstract) syntagmatic and paradigmatic relations. Abstract syntagmatic relations link units of one abstract stratum into segments which are units of a hierarchically higher abstract stratum, whereas abstract paradigmatic relations are those relations between units of one abstract linguistic stratum which are not abstract syntagmatic relations. It is worth noting that two types can also be distinguished among the abstract paradigmatic relations: relations of the first type, which make it possible to form new abstract units (e.g. the relation of homophonemity between phones, whose equivalence classes are phonemes), and the remaining abstract paradigmatic relations (e.g. complementary distribution of phones).

Apart from syntagmatic and paradigmatic relations, the structure of language is constituted by other relations (thus, syntagmatic and paradigmatic relations do not exhaust the entire spectrum of linguistic relations). The above definitions indicate that such relations (being neither syntagmatic nor paradigmatic) always occur between units belonging to different linguistic strata. It makes sense, therefore, to call them INTERSTRATAL relations. An example of an interstratal relation is a three-argument relationship between phonemes, phonemic environments, and phones, which occurs when a given phoneme is realised as a given phone in the appropriate phonemic environment (for instance, in the Polish language the phoneme [n] in the environment before [k] is realized as the phone [ŋ]).

If we consider relations between segments of an utterance and objects like situational contexts, language users, etc. as linguistic relations, they will also be interstratal relations of a kind. In this case, sets of situational contexts, language users, etc. will be abstract linguistic strata.

The above classification of linguistic relations is summarised in the following table.

	relations defined between units of the same linguistic stratum	relations between units belonging to different linguistic strata (interstratal)		
	syntagmatic (binding linguistic units into larger segments)	paradigmatic (the remaining, i.e. non-syntagmatic relations defined between units of a linguistic stratum)		
		of the 1st type (creating abstract units)	of the 2nd type (the remaining ones. i.e. those, which do not belong to the 1st type)	
concrete (binding segments of an utterance)	e.g. the succession of sounds in the speech process	e.g. homophony	e.g. facultative variation of sounds	e.g. the mereological relation of being a part
abstract (between abstract units)	e.g. agreement, text cohesion	e.g. homophonemity between phones	e.g. hyponymy	e.g. realization of a phoneme as a given phone in the appropriate environment

The entire body of linguistic units and linguistic relations creates the LANGUAGE SYSTEM. In the following section of this article, we will discuss the feasibility of a comprehensive presentation of a language system. This presentation will be demonstrated in the simplest manner possible, that is by means of the elementary resources of formal logic. We assume that the Reader is familiar with the basic knowledge regarding set and relational calculus. We shall use the standard set-theoretical notation.

3. The discussed feasibility of presenting the language system as the body of linguistic units and relations makes use of the concept of a GENERAL SYSTEM, introduced in (Pogonowski 1979: part 3). To define this concept, it is necessary to recall the construction of the class V_X over the atom set X (cf. e.g. Shoenfield 1967: ch. 9.1, Barwise 1975: 42). This construction makes use of transfinite induction:

$$V_X^0 = X$$
$$V_X^{a+1} = P(X \cup V_X^a)$$
$$V_X^\lambda = \bigcup_{a \in \lambda} V_X^a \text{ for the limit ordinal numbers } \lambda$$
$$V_X = \bigcup_{a \in On} V_X^a$$

(symbol P signifies the operation of the power set, that is the operation of creating a family of all subsets, while On is the class of all ordinal numbers).

Set X is any set; its elements may be atoms (i.e. objects which are not sets) or other sets. The class V_X therefore equals the totality of sets which can be constructed with family X as the point of departure.

DEFINITION 1. Let $\mho$ be any family of sets and let $U = \bigcup \mho$. We say that Σ is a GENERAL SYSTEM GENERATED by $\mho$ if $\Sigma = (\mho, C)$, where $C \subseteq V_U$. Family $\mho$ is the family of LEVELS of system Σ, set U is the set of OBJECTS of system Σ, while family C is the SIGNATURE of Σ.

The intuitive sense of the concept of the general system $\Sigma_0 = (\mho, C)$ is as follows: signature C is the family of relationships between the objects of U grouped into levels of U. It follows from the definition that the signature of the general system may contain relations between objects of that system, relations between sets of objects, relations between relations, etc. This is a consequence of the extensional approach — the signature of the general system may contain any set-theoretical construct obtained from the objects of that system.

Due to its high level of generalisation, the concept of a general system has a very wide range of applications. For instance, all relational structures, many-sorted structures (e.g. vector spaces, abstract automata), structures with an infinite number of relations (e.g. topological spaces), etc. are all general systems as understood in Def. 1. Formal languages, as set-theoretical constructs over a suitable alphabet, can also be represented as general systems. Of course, by adding various conditions to the definition of a general system, we can obtain many types of more concrete systems. Without delving into this issue, let us only point out the feasibility of applying the concept of a general system in linguistics in order to present the language system as a general system as understood in Def. 1.

The simplest method of presenting the language system as a certain general system is to assume all its linguistic strata to be its levels, and all linguistic relations as its signature. Then, of course, precisely all linguistic units will be objects of our general system.

Let us introduce another useful concept linked with the concept of the general system.

DEFINITION 2. Let $\Sigma = (\mho, C)$ be a general system generated by $\mho$. Let $\mho^*$ signify the family of all finite Cartesian products of the sets from $\mho$. Let us define the function $c_\Sigma : \mho^* \to P(C)$ by $c_\Sigma(K) C \cap P(K)$ for any $K \in \mho^*$. The function c_Σ is called the RELATIONAL CHARACTERISTICS of

system Σ.

Note that the set of values of the relational characteristics of a given general system is composed of all relations between objects of that system.

Let us return to the language system treated as the general system $\Sigma_0 = (\mho_0, C_0)$, that is, let us interpret $\mho_0$ as a set of all linguistic strata and as the family of all linguistic relations. Let e.g. $\mho_0 = \{U_i : i \in I\}$, where I is the appropriate set of indices. Then $\bigcup_{i \in I} U_i$ is the set of all linguistic units. For any set X, let X^n be the n-th Cartesian power of X. Let us consider the following structure for any linguistic stratum U:

$$(5)\ (U_i, \bigcup_n c_{\Sigma_0}(U_i^n))$$

The relational structure (5) is a sub-system of a language system composed of a set of linguistic units belonging to stratum U_i and all linguistic relations defined between units of this stratum. Let us note that the set $\bigcup_n c_{\Sigma_0}(U_i^n)$ contains exclusively syntagmatic and paradigmatic relations. In particular, is the set of all one-argument linguistic relations defined on U_i, $c_{\Sigma_0}(U_i \times U_i)$ is a set of all two-argument relations defined on U_i, etc. In the form of structure (5), it is possible to represent, for instance, the set of all phones with the relations between phones (homodistribution, facultative variation, complementary distribution, etc.) or the set of all words with the relations of hyponymy, synonymy, antonymy, semantic fields, etc.

If U_i and U_j are two different linguistic strata, then $c_{\Sigma_0}(U_i \times U_j)$ is the set of all (two-argument) linguistic relations between units of the stratum U_i and units of the stratum U_j. For instance, if U_i is the phone stratum, and U_j is the phoneme stratum, then $c_{\Sigma_0}(U_i \times U_j)$ contains all relations between phones and phonemes.

It is easy to notice that the family:

$$(6) \bigcup_{i \in I} \bigcup_n c_{\Sigma_0}(U_i^n)$$

is the set of all syntagmatic and paradigmatic linguistic relations. All inter-stratal relations create the following set:

$$(7)\ \bigcup_{K \in \mho_0^+} c_{\Sigma_0}(K)$$

where $\mho_0^+$ denotes the family of all Cartesian products of linguistic strata

which are not Cartesian powers of some linguistic stratum, that is:

$$(8) \ \mho_0^+ = \mho_0^* - \bigcup_{i \in I} \bigcup_n U_i^n$$

Of course, sets (6) and (7) give in sum the signature C_0:

$$(9) \ C_0 = \bigcup_{i \in I} \bigcup_n c_{\Sigma_0}(U_i^n) \cup \bigcup_{K \in \mho_0^+} c_{\Sigma_0}(K)$$

Finally, let us note that the following equality holds in the interpretation accepted here:

$$(10) \ C_0 = \bigcup_{K \in \mho_{0*}} c_{\Sigma_0}(K)$$

(this form of equality not necessarily holds for every general system).

Properties of the language system treated as a general system may be examined with the aid of the resources of formal logic. In particular, by means of the system-theory apparatus, it is possible to describe various issues regarding, for instance, sub-systems of the language system, morphisms between those sub-systems, further classifications of linguistic relations with respect to their general characteristics, etc. Finally, let us observe that the approach described in this section of the study is quite universal — other possible representations of the language system can all be described in the form of appropriate general systems.

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