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# AN OUTLINE OF FORMAL SEMIOTICS

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## 1. Introduction

The aim of this article is to present a fragment of a system of formal semiotics. It is a part of a greater whole — a uniform system which embraces, among other things, the issues of the structure of a sign, of the relation between signs and real objects, of the semantics of signs and perception of signs, of their pragmatic role.

The discussion will focus on issues of formal semantics of signs; the proposed theory will be based on concepts of fuzzy set theory.

## 2. Basic primitive concepts

The easiest way to present the formal theory will be to start with the simplest version of the system and then enrich it if necessary.

Accordingly, we will first introduce a system comprising three primitive concepts:

(1) $<S, M, \rho>$,

where $S$ and $M$ are sets of SIGNS and their MEANINGS, respectively, and $\rho \subset S \times M$ is a relation which connects signs $s \in S$ with meanings $m \in S$. The symbol $s\rho m$ will be used for: "$m$ is the meaning of $s$."

Concepts $S$, $M$, and $\rho$ have an internal structure, which will be introduced in subsequent sections. Let us denote:

(2) $\rho(s) = \{m \in M\colon s\rho m\}$,
(3) $\rho^{-1}(m) = \{s \in S\colon s\rho m\}$.

If $\rho(s) \neq \varnothing$, the sign $s$ shall be called MEANINGFUL or INTERPRETABLE; for given $s_1$, $s_2 \in S$, (partial) SYNONYMY will be defined by the requirement $\rho(s_1) \cap \rho(s_2) \neq \varnothing$, that is, by demanding that there should be at least one meaning shared by $s_1$ and $s_2$. Synonymy is FULL if $\rho(s_1) = \rho(s_2)$.

A special case of synonymy, where $\rho(s_1) \subset \rho(s_2)$, shall be called HYPONYMY. Finally, a general case where $\rho(s_1) \cap \rho(s_2) \neq \varnothing$, $\rho(s_1) \setminus \rho(s_2) \neq \varnothing$, and $\rho(s_2) \setminus \rho(s_1) \neq \varnothing$, shall be called EQUIPOLLENCE. Thus two signs are equipollent if they have some common meanings, yet each of them have some additional meanings, not possessed by the other.

The relations introduced above have the following properties:

*Theorem*. The relation of complete synonymy is an equivalence, while partial synonymy is reflexive and symmetrical, but not transitive. The hyponymy relation is reflexive and transitive, but not symmetrical. Finally, equipollence is symmetrical, but neither reflexive, nor transitive.

By employing sets of form (3) one can say that a meaning $m$ is EXPRESSIBLE if $\rho^{-1}(m) \neq \varnothing$. Next, if $\rho^{-1}(m_1) \cap \rho^{-1}(m_2) \neq \varnothing$, then every sign $s$ belonging to this intersection (given that $m_1 \neq m_2$) will be called EQUIVOCAL; it has at least two different meanings, $m_1$ and $m_2$.

## 3. Extension of the system

We will now enrich system (1) by introducing: (a) a division of signs into categories, (b) structural elements of the set of meanings $M$, in the form of a relation describing 'distances' between meanings, and (c) a 'fuzziness' of the relation $\rho$.

Accordingly, the system of primitive concepts will take the form of:

(4) $<S, \mathscr{F}, M, \tau, f>$,

where $S$ and $M$ symbolize the same sets as above, whereas $\mathscr{F}$ is a class of divisions of $S$, so that each element $F \in \mathscr{F}$ is a family of sets $S_1$, ..., $S_n$ such that:

(5) $S_i \cap S_j = \varnothing$, for $i \neq j$,

$$(6)\ \bigcup_i S_i = S.$$

Elements of each division from $\mathscr{F}$ shall be called taxonomic categories of signs.

For two given divisions, $F$ and $F'$, into sets $S_1$, ..., $S_n$, and $S'_1$, ..., $S'_m$, it is possible to define their intersection $F \cap F'$ in the following way:

(7) $F \cap F' = \{S_i \cap S'_j,\ i = 1, ..., n, j = 1, ..., m\}$.

The easiest way to define the union of two divisions, $F \cup F'$, will be to make use of equivalence relations. Namely, each division can be associated with a relation $\sim_F$, defined by a requirement that $s \sim_F s'$ if and only if $s$ and $s'$ belong to the same set from the division $F$. Conversely, each equivalence relation defines a division into its own equivalence classes.

Now let $\sim_F$ and $\sim_{F'}$ be two equivalence relations corresponding to divisions $F$ and $F'$. Then $F \cup F'$ is defined as a division corresponding to the relation which is a transitive extension of the sum of relations $\sim_F$ and $\sim_{F'}$, that is, $x \sim_{F \cup F'} y$ if:

(8) $\exists s_1, ..., s_r : (x \sim_F s_1 \vee x \sim_{F'} s_1) \wedge (s_1 \sim_F s_2 \vee s_1 \sim_{F'} s_2) \wedge ... \wedge (s_r \sim_F y \vee s_r \sim_{F'} y)$.

Then we have:

*Theorem*. Operations $\cap$ and $\cup$ satisfy the following laws of idempotence:

(9) $(F \cap F') \cap F = F \cap F'$,
(10) $(F \cup F') \cup F = F \cup F'$.

As for the other concepts of system (4), $\tau$ is a quaternary relation in $M$ with the following intended interpretation. If $(m_1, m_2, m_3, m_4) \in \tau$, which will be symbolized as $(m_1, m_2)\tau(m_3, m_4)$, then the 'difference' (or a subjectively assessed 'distance') between meanings $m_1$ and $m_2$ is greater than the difference between meanings $m_3$ and $m_4$.

It will be assumed that the relation $\tau$ satisfies the following conditions; for every $m_1$, $m_2$, $m_3$, $m_4$, $m_5$, $m_6$:

*Postulate* 1. If $(m_1, m_2)\tau(m_3, m_4)$, then $(m_1, m_2)\tau(m_3, m_4)$ and $(m_1, m_2)\tau(m_4, m_3)$.

*Postulate* 2. If $(m_1, m_2)\tau(m_3, m_4)$ and $(m_3, m_4)\tau(m_5, m_6)$, then $(m_1, m_2)\tau(m_5, m_6)$.

*Postulate* 3. $(m_1, m_2)\tau(m_3, m_3)$.

*Postulate* 4. If $(m_1, m_1)\tau(m_2, m_3)$, then $m_2 = m_3$.

Thus postulate 1 states that distances are symmetrical with respect to their arguments; postulate 2 says that the relation of distance comparison is transitive; postulate 3 asserts that the distance between two identical meanings equals 0; finally, postulate 4 declares that zero distance implies that the meanings must be identical.

Before formulating the last postulate for the relation $\tau$ we will discuss the last primitive concept of system (4) — the function *f*. It is the 'fuzziness' of the relation $\rho$ from system (1); formally, $f$ is a function:

(11) $f : S \times M \rightarrow [0, 1]$,

where $f(s, m)$ represents the degree to which $s$ has the meaning $m$.

In the special case in which $f$ only takes 0 and 1 as values, we have:

(12) $\rho = \{(s, m) : f(s, m) = 1\}$

In general, for any $0 \leqslant \alpha \leqslant 1$, let us define a relation:

(13) $\rho_\alpha = \{(s, m) : f(s, m) \geqslant \alpha\}$,

such that $\rho_\alpha$ is a (non-fuzzy) relation in $S \times M$ induced by the relation $f$ and the level $\alpha$. Note that:

*Theorem*. If $\alpha \leqslant \beta$, then $\rho_\alpha \supset \rho_\beta \supset \rho$.

We are now in a position to formulate a postulate which connects the fuzzy relation $f$ with the relation $\tau$.

*Postulate* 5. Suppose that $\alpha > \beta$, $s\rho_\alpha m_1$, $s\rho_\alpha m_2$, and it is not the case that $s\rho_\alpha m_3$. If $s\rho_\beta m_3$, then $(m_2, m_3)\tau(m_1, m_2)$ or $(m_1, m_3)\tau(m_1, m_2)$.

This postulate describes the following property. Suppose that a sign $s$ expresses two meanings, $m_1$ and $m_2$, at least to the degree $\alpha$. Let us also assume that $s$ has another meaning, $m_3$, but expressed to a lesser degree, $\beta$. In such a case the distance between $m_1$ and $m_2$ (between 'stronger' meanings) is smaller than from one of these meaning to $m_3$.

Let us fix a certain level $\alpha$ and examine the connections between relations $\rho_\alpha$ and divisions of the set $S$ into taxonomic categories.

For a fixed division $F = \{S_1, ..., S_n\}$, let us define:

$$(14)\ \rho_\alpha^{(k)} = \rho_\alpha \cap (S_k \times M)$$

For each meaning $m$, let $s_k(m)$ stand for the sign in $S_k$ which expresses $m$ to the highest possible degree, i.e., which meets the condition:

$$(15)\ f(s_k(m),\ m) = \sup_{s \in S_k} f(s, m).$$

(It is assumed, for simplicity, that the supremum is achieved.)

Let $\alpha_k(m,F)$ denote the common value of equation (15). Then we get:

*Theorem.* For every $\alpha \leqslant \alpha_k(m,F)$, $(s_k(m),\ m) \in \rho_\alpha^{(k)}$.

The vector:

(16) $(\alpha_1(m,F), \alpha_2(m,F), ..., \alpha_n(m,F))$

will be called SPECTRUM of the meaning $m$. It expresses the maximum degrees to which one can efficiently express $m$ by means of particular categories of the division $F$.

Clearly, $\max_k \alpha_k(m, F)$is independent from a given division $F$; however, let us denote the average level of expressing $m$ by means of signs of different categories of the division $F$:

$$(17)\ d(m, F) = \frac{1}{n} \sum_{i=1}^{n} a_i(m, F).$$

Then we obtain:

*Theorem*. For any divisions $F$, $F' \in \mathscr{F}$:

(18) $d(m, F \cap F') \leqslant \min(d(m, F), d(m, F')) \leqslant \max(d(m, F), d(m, F')) \leqslant d(m, F \cup F')$.

The theorem expresses an interesting feature that fragmentation of a division into sign categories decreases the average degree of 'expressibility' of a meaning $m$ by means of signs of different types.

For a proof, suppose that $F$ and $F'$ are divisions into $S_1, ..., S_n$ and $S'_1, ..., S'_r$, respectively. Then:

$$(19)\ d(m, F \cap F') = \frac{1}{rn} \sum_{i=1}^{n} \sum_{j=1}^{r} a_{ij}(m, F \cap F^{'}),$$

where:

$$(20) a_{ij}(m, F \cap F^{'}) = \sup_{s \in S_i \cap S'_j} f(s, m).$$

But:

$$\sum_{j=1}^{r} a_{ij}(m, F \cap F^{'}) \leqslant r \sup_{s \in S_i} f(s, m) = r a_i(m, F),$$

and by substituting (19) we obtain the first inequality of the theorem. The remaining inequalities are proven in an analogous way.

## 4. Sign composition

We will now add yet another primitive concept to the discussed system (4), namely, the notion of sign composition.

Thus, if $s_1, s_2 \in S$, then $s_1 o s_2$ will represent a sign composed of $s_1, s_2$.

Of course, not every composition of signs is possible, and the operation $o$ is not always definitely interpretable. We will assume that the relation $o$ is a primitive concept of the system, i.e., there is a fixed set of pairs $(s_1, s_2)$ such that $s_1 o s_2 \in S$, and in what follows we will tacitly assume that the symbol $o$ will be applied only to those pairs of signs for which the relation $o$ is defined.

A typical example of the relation $o$ — for the category of signs that are notations of strings of words — is their concatenation.

Consider signs $s_1$ and $s_2$ together with their composition $s = s_1 o s_2$. For a fixed $\alpha$ we will have the following sets:

(21) $\rho_\alpha(s_1) = \{m : s_1\rho_\alpha m\}$, $\rho_\alpha(s_2) = \{m : s_2\rho_\alpha m\}$, $\rho_\alpha(s) = \{m : s\rho_\alpha m\}$,

We can now put forward the following definitions:

Signs $s_1$ and $s_2$ are ORTHOGONAL if:

(22) $(\forall\alpha)$: $\rho_\alpha(s) = \rho_\alpha(s_1) \cup \rho_\alpha(s_2)$

.

Each meaning $m$ such that $m \in \rho_\alpha(s_1) \cup \rho_\alpha(s_2)$ and $m \notin \rho_\alpha(s)$ shall be called $\alpha$-INHIBITED in sign composition.

Conversely, if $m \in \rho_\alpha(s)$, while $m \notin \rho_\alpha(s_1) \cup \rho_\alpha(s_2)$, then $m$ is $\alpha$-GENERATED in composition of $s_1$ and $s_2$.

These definitions are relative with respect to a given level $\alpha$ of meaning representation. If we allow for various levels of representation, we can introduce the following definitions (cf. Nowakowska 1976).

Suppose that $\alpha < \beta$ and $m \in \rho_\alpha(s_1) \cup \rho_\alpha(s_2)$, but $m \notin \rho_\beta(s_1) \cup \rho_\beta(s_2)$. If $m \in \rho_\beta(s)$, then $m$ is $(\alpha, \beta)$-SUPPORTED by the composition.

Conversely, if $m \in \rho_\alpha(s_1) \cup \rho_\alpha(s_2)$ and $m \notin \rho_\alpha(s)$, while $m \in \rho_\beta(s)$ for $\beta < \alpha$, then $m$ is $(\alpha, \beta)$-INHIBITED.

## 5. Objects and signs

In this and in the subsequent section we will put forward an outline of a theory of the connection between signs, objects represented by those signs, sign perception, and the reflection of this perception in the form of a verbal copy of an object.

The starting point will be a formal representation of an object as a relational structure of the form:

$<P,\ A,\ q,\ \mathscr{R}>$,

where $P$ is a set of elements interpreted as PARTS of objects, $A$ is a set of ATTRIBUTES, $q$ is a relation in $P \times A$ which assigns attributes to parts of an object, and $\mathscr{R} = \{R_1,\ R_2,\ ...\}$ is a family of relations in $P$.

Generally, if an object is represented in the form of a configuration of graphic signs, then — with the exception of purely conventional signs — there is a certain correlation between the structure of a sign and the structure of an object. Namely, a sign, say $s$, can also be interpreted as an object, i.e. as a relational structure of the form $s = <P_z,\ A_z,\ q_z,\ \mathscr{R}_z>$ with the same interpretation as before (i.e. a set of parts, a set of attributes of these parts, etc.).

If a sign represents an object $T$, then there is a function $\varphi$ mapping the relational configuration of the sign onto the relational configuration of the object, which preserves (at least some of) the connections. Without going into technical details, let $P_s^\varphi \subset P,\ A_s^\varphi \subset A,\ q_s^\varphi \subset q,\ R_s^\varphi \subset R$denote those parts, attributes, etc. which are reflected in the sign $s$.

Generally, the more elements of the above sets are reflected in $s$, the more iconic $s$ is, and one could be tempted to build a 'iconicity index' of $s$.

As it happens, it is possible to proceed in a slightly different way, by considering not only which fragments from the set $P$ are in the set $P_z^\varphi$, but also how important they are. Namely (Nowakowska 1967), one can assign to particular parts $x \in\ P$ numbers $w(x)$ representing the degree of IMPORTANCE of these parts in recognizing the object. These numbers, called weights henceforth, are formally defined in terms of coalition theory, and more specifically, by means of the Shapley—Shubik power index, which measures the powers of members of legislative bodies (Shapley and Shubik 1954). One can indicate an empirical procedure which leads — at least in the case of simple objects — to assigning those weights.

Understandably, a sign can apply not only to a single object, but — more generally — to a situation, that is, to a configuration of a certain number of objects. A description must distinguish a set of objects, every one of which is a relational structure presented above, and certain relations characterizing mutual connections between these objects. Such an account leads to a kind of algebra of situations and allows us to analyze correlations between the structure of a situation and the structure of its verbal copy (description); an outline of this theory can be found in the next section.

At this point, it is worth considering signs of a different kind, namely signs concerning certain actions. In this case the adequate formalization is provided by the theory of action (Nowakowska 1973), which can be — very roughly — represented in the form of a structure:

(24) $<D, L, S, R>$,

where $D$ is a set of elementary actions (specific to a given situation), $L \subset D^*$ is a class of sequences of elements of the set $D$, i.e. a subset of the monoid over $D$. Sequences from $L$ are interpreted as acceptable strings of actions, and $L$ itself is dubbed a language of actions, due to the analogy with the natural language, which is a class of strings of words (or natural languages which are classes of strings of symbols from a certain alphabet). Next, $S$ is a set of the results of actions, and $R$ is a relation linking together the action sequences from $L$, results from $S$, and the times of their occurrence.

This formal structure has turned out to be unexpectedly rich in theoretical consequences and interpretive possibilities, allowing us to define a great number of concepts crucial for describing actions, such as attainability and its various types, moments of decision, goals, means of attaining them, effectiveness of actions, praxeological character of actions, etc.

In the case of the semantics of signs, this structure can be exploited in the following way. Let $L$ designate a language of actions specific to a given situation or class of situations, and let $\Phi$ denote a class of motivational operators (cf. Nowakowska 1973), such as "I should," "I want," "It is worth," etc. Then, for a given sign $s$, one can consider a relation:

(25) $Q(s) \subset \Phi \times L$,

where $(g, u) \in Q(s)$, $g \in \Phi$, $u \in L$ means that the sign $s$ connects the operator $g$ with a sequence of actions $u$.

It is then natural to consider the following sets:

(26) $\Phi(s) = \{g \in \Phi : (g, u) \in Q(s)\}$ for some $u \in L\}$,
(27) $L(s) = \{u \in L : (g, u) \in Q(s)\}$ for some $g \in \Phi\}$,

So $\Phi(s)$ characterizes a TYPE of sign from a pragmatic point of view; the categories would be instructions, commands, prohibitions, etc. corresponding to operators such as "It is worth," "One ought to," "It is necessary that," etc. for instructions, and similarly for other types. On the other hand, $L(s)$

can be called ”a language of actions of the sign $s$;” in fact, it is the set of sequences of actions which $s$ applies to — that is, which are commanded, prohibited, etc. by the sign.

Signs $s$ and $s'$ such that $L(s) = L(s')$, while $\Phi(s)$ and $\Phi(s')$ contain opposite operators, form a natural oppositional pair (a typical example would be the stop sign and a prohibition of coming to a halt).

## 6. Algebra of situations and verbal copies

As a final point, we will sketch a theory of signs of a special form, namely, verbal copies.

As pointed out above, a situation can be equated with a configuration (relational structure) of objects. Generally, in a description one can distinguish a set of attributes expressing relevant properties and the corresponding sets of values $W_1$ (perhaps qualitative in character; these values will be generally called descriptors).

A complete description of a situation will be a system:

(28) $< W_1, ..., W_n, E^* >$,

where the meaning of $E^*$ will be explained below.

By an ELEMENTARY situation we will understand a vector:

(29) $\bar{V} = (V_1, \ldots, V_n)$

where $V_i \subset W_i$, for $i = 1, ..., n$.

Let $E$ designate the set of all elementary situations. If $\bar{V} = (V_1, \ldots, V_n)$ and $\bar{V}' = (V'_1, \ldots, V'_n)$, then the intersection and union of situations $\bar{V}$ and $\bar{V}'$ is described as:

$$(30) \bar{V} \cdot \bar{V}' = (V_1 \cap V'_1, \ldots, V_n \cap V'_n),$$

$$(31) \bar{V} + \bar{V}' = (V_1 \cup V'_1, \ldots, V_n \cup V'_n).$$

Then the following theorem is true:

*Theorem.* The class $E$ is closed under the operations of intersection and union.

Furthermore:

$$(32)\ \bar{V}\cdot\bar{V}=\bar{V},\ \bar{V}+\bar{V}=\bar{V}(idempotence)$$

$$(33)\ \bar{V}\cdot\bar{V}^{'}=\bar{V}^{'}\cdot\bar{V},\ \ \bar{V}+\bar{V}'=\bar{V}'+\bar{V}\ (commutative\ property)$$

$$(34)\ \bar{V}\cdot(\bar{V}^{'}+\bar{V}^{''})=\bar{V}\cdot\bar{V}^{'}+\bar{V}\cdot\bar{V}^{''},\ \bar{V}+(\bar{V}'\cdot\bar{V}^{''})=(\bar{V}+\bar{V}')\cdot(\bar{V}+\bar{V}^{''})(distributive\ property)$$

The relation of inclusion of situations, $\bar{V}\subset\bar{V}'$is defined by the requirement that $\bar{V}\cdot\bar{V}^{'}=\bar{V}$ .

The last primitive concept of system (28), namely $E^*$, is a certain subset of $E$, interpreted as the situations which actually take place.

It is assumed that $E^*$ has the following features:

*Assumption.*

(35) $\bar{V},\bar{V}\in E^*\Rightarrow\bar{V}\cdot\bar{V}'\in E^*$
(36) $E^*$ is non-empty
(37) $(\forall i)(\exists\varnothing\neq U_i\subset\ W_i)$: $(W_1,\ ...,\ W_{i-1},\ U_i,\ W_{i+1},\ ...,\ W_n)\notin\ E^*$
(38) $(\forall i)(\forall V_1,\ ...,\ V_{i-1},\ V_{i+1},\ ...,\ V_n)$: $(V_1,\ ...,\ V_{i-1},\ \varnothing, V_{i+1},\ ...,\ V_n)\in\ E^*$

This assumption means the following. According to relation (35) an intersection of two situations which actually take place is also a situation which actually takes place. Condition (36) determines that some situation occurs. According to relation (37), for every attribute, there are descriptor values which fail to occur in reality (so this assumption eliminates trivial attributes). Finally, assumption (38) states that each attribute has a certain descriptor (that is, some descriptor describes what is actually the case).

We are now in a position to define the concept of minimal and maximal situation that occurs in reality, say $\bar{V}_{min}$ and $\bar{V}_{max}$, by means of the relations:

$$(39)\ \forall\bar{U}\in E^*:(\bar{U}\subset\bar{V}_{min}\Rightarrow\bar{U}=\bar{V}_{min}),$$

(40) $\forall \bar{U} \in E^* : (\bar{U} \supset \bar{V}_{max} \Rightarrow \bar{U} = \bar{V}_{max})$.

We will prove the following theorem.

*Theorem.* There is exactly one minimal situation.

For a proof, assume that $\bar{V}^{(1)}_{min}$ and $\bar{V}^{(2)}_{min}$ satisfy (39), and let $\bar{U} = \bar{V}^{(1)}_{min} \cdot \bar{V}^{(2)}_{min}$ . Then $\bar{U}$ is contained both in $\bar{V}^{(1)}_{min}$ and in $\bar{V}^{(2)}_{min}$. This intersection can have no empty coordinate, because otherwise it would not belong to $E^*$, contrary to (35). Hence $\bar{U} \in E^*$, and it must be the case that $\bar{U} = \bar{V}^{(1)}_{min} = \bar{V}^{(2)}_{min}$.

The situation $\bar{V}_{min}$ will be called THE TRUE STATE OF AFFAIRS. The maximal state of affairs can be equated with the effect of various bonds by virtue of which some states (values of some attributes) rule out combinations of other values.

This account of situations allows us to analyze dynamic aspects of changes of situations (cf. Nowakowska 1973). For this purpose it must be assumed that the set $E^*$ changes in time. Therefore, the true state of affairs $\bar{V}_{min}$ is also a function of a time $t$. By considering the set of all 'histories' $\bar{V}_{min}(t)$ we can define the concept of EVENT as a subset of a history. Then, by combining histories with actions which influence these histories, we obtain a systematic account of ACTION and CONTROL, where the GOAL is defined by a configuration of events (cf. also Nowakowska 1976).

Let us now return to the main topic, that is, to the issue of verbal copies. We are in a position to introduce the concept of the 'LANGUAGE OF DESCRIPTION', by considering, for each attribute, a certain class $L_i$ of subsets of the set of descriptors $W_i$. Namely, these are subsets of $W_i$ which have their own NAME. With respect to classes $L_i$ we will assume that:

*Assumption.*

(41) $U \in L_i \Rightarrow W_i \setminus U \in L_i$,

that is to say, the class $L_i$ is closed under the operation of completing (yet it is not required that it be closed under the conjunction or alternative).

For instance, if the attribute in question is colour, then the elements of $W_i$ are descriptors such as "white," "black," etc. Some subsets of $W_i$ have their own names, like "black-and-white," "bicoloured," etc. which are expressed by corresponding sets of descriptors (i.e. by subsets of $W_i$).

A VERBAL COPY is a conjunction of sentences of the form "$x$ is $V_i$," where $V_i \in L_i$. A copy is said to be FAITHFUL if all its sentences have the following property:

(42) $V_i \in L_i \cap i(E^*)$,

where

(43) $i(E^*) = \{U_i : \exists V_1, ..., V_{i-1}, V_{i+1}, ..., V_n$ such that $(V_1, ..., V_{i-1}, U_i, V_{i+1}, ..., V_n) \in E^*\}$.

Thus $i(E^*)$ is a projection of $E$ onto the $i$-th coordinate.

If $C$ is a verbal copy, let $C_i$ denote all sentences in $C$ referring to the $i$-th attribute; let them be sentences "$x$ is $V_i^{(1)}$," ..., "$x$ is $V_i^{(m_i)}$." Now we can introduce the following definition. A copy is EXACT if it satisfies the condition:

$$(44)(\bigcap_{i=1}^{m_1} V_1^{(i)}, \ldots, \bigcap_{i=1}^{m_n} V_n^{(i)}) = \bar{V}_{min}.$$

In other words, an exact copy is a copy which unambiguously specifies the value of each attribute.

Whether faithful copies exist, or not, is decided by how rich languages $L_i$ are. The following theorem holds.

*Theorem.* A faithful copy of each situation exists if and only if:

$$(45)\ (\forall i)(\forall w \in W_i)(\exists U_i, ..., U_r \in L_i) : \bigcap_{i=1}^{r} U_i = \{w\},$$

that is, if every value of an attribute (a descriptor) is expressible as a conjunction of expressions of $L_i$.

The above formal notions concerning properties of verbal copies, together with the concept of the weight of fragments, described in the preceding section, make it possible to formulate empirically testable hypotheses about mechanisms of generating verbal copies.

**Bibliography**


1. Nowakowska, Maria (1967) ”Quantitative Approach to the Dynamics of Perception.” *General Systems* 12: 81—95.

2. Nowakowska, Maria (1973) *Language of Motivation and Language of Actions*. The Hague: Mouton.

3. Nowakowska, Maria (1976) ”Action Theory: Algebra of Goals and Algebra of Means.” *Design Theories and Methods* 10: 97—102.

4. Nowakowska, Maria (1979) ”Komunikacja werbalna i niewerbalna jako język wielowymiarowy.” *Studia Semiotyczne* 9: 181—196.

   Nowakowska, Maria (1983) ”Cybernetics in the Social Sciences.” In *Cybernetics: Theory and Applications*, Robert Trappl (ed.), 177—234. Washington (DC): Hemisphere.

5. Shapley, Lloyd S. and Martin Shubik (1954) ”A Method for Evaluating the Distribution of Power in a Committee System.” *American Political Science Review* 48[3]: 787—792.