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PRO-FORMS INSTEAD OF VARIABLES AND OPERATORS

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I

Inquiries into the logical analysis of a natural language tend to be inspired by the philosophical problems that a natural language generates; less commonly, it is the case because of a conviction that a natural language, despite even more widespread applications of formalized languages, is an indispensable tool of accumulating and transmitting information of cognitive significance.

In the research that stems from interest in a natural language as the language of passing on that which is scientifically valuable, two styles can be distinguished. The first one is about interpreting the expressions and syntactic forms of a natural language by way of indicating the manner of their translation into a standard formalized language. Apparently, at the root of this method of investigating natural language lies a conviction that the grammatical peculiarities are something irrelevant, non-functional and, essentially, it is the translation into a standard formalized language that reveals a true syntactic structure of natural language expressions and the accompanying semantic interdependencies. This method is illustrated by the book by Hans Reichenbach *Elements of Symbolic Logic* as well as one of Roman Suszko's early works *Zarys elementarnej składni logicznej*. The other style can be exemplified by Adelina Morawiec's *Podstawy logiki nazw*, which presents the conception of the logic of names, put forward by Suszko and elaborated on in a seminar for the Polish Academy of Sciences (PAN) Chair of Logic in 1966. It is about the creation of formalized languages that constitute RATIONAL RECONSTRUCTIONS of passages from a natural language. The starting point here tends to be selecting from a natural language some types of expressions and syntactic forms that can be filled by these expressions. The syntactic rules are arrived at by the analysis of these forms. In the same way, we reach the formulation of the semantic interdependencies between expressions. The rational character of reconstruction is about ignoring these peculiarities of natural language which from the standpoint of a cognitive purpose can be seen as non-functional, and it is also about the assumption of the principle of unlimited construction (Suszko 1957: 40-41). A rational reconstruction of passages from a natural language can be more or less close to the original model, that is, more or less adequate. In order to try and obtain as adequate a reconstruction as possible, one should be careful in qualifying the phenomena occurring in a natural language as non-functional. It ought to be remembered that any graphic natural language has a sonic counterpart for aural perception. Accounting for this fact reveals the functionality of a number of features of a natural language.

The two research styles outlined above do not preclude each other. Indicating a way of translating expressions from a passage of a natural language into a standard formalized language can be a preparatory action for the reconstruction of this passage in the form of a non-standard formalized language.

The method of creating rational reconstructions of passages from a natural language does not stray from the commonly accepted principles of formalized language construction and is about 1) establishing the resource of simple vocabulary, 2) indicating the rules of creating complex expressions, with sentences in particular, 3) describing these constructions in semantic terms, that is, establishing the interrelationships between the objective correlates of simple expressions and the same kinds of correlates of complex expressions. The implementation of the last point presupposes the determination of a class of language models and it leads directly to the definition of the notion of a true sentence in a given model, which allows for the definition of a number of semantic notions, including the notion of logical consistency. Defining the notion of logical consistency is the aim of the whole project as it is the task of a logician who investigates a language to discover the logic inherent in it, that is, the function of logical consistency, determined in the set of sentences of this language.

It would be unreasonable to believe in the possibility of making a rational reconstruction of the whole natural language. This undertaking is made futile by the lack of strict resolutions on what is and what is not a correctly constructed sentence of this language. A natural language is a domain for incessant creativity of its users and thus any grammar can be adequate only vis-a-vis some specific stage of the development of language, and only in approximation. This is not the only difficulty, though. Another major obstacle is that the methods and concepts used in logic do not allow moving beyond SEMANTICALLY self-sufficient PROPOSITIONS. By using this *ad hoc* term I mean sentences whose interpretation is not dependent on circumstances that are external to language. Semantically self-sufficient sentences are utterances whose comprehension only needs from the addressee the acquisition of the meanings of words as well as the grammatical and semantic rules proper to a language. Comprehending these propositions does not depend on whether the recipient understands the circumstances that accompany their production and nor does it depend on their knowledge or inferential ability.

Semantically self-sufficient sentences are not rare in a natural language; moreover, for any natural language proposition whose sense is definite in given circumstances, one can demonstrate an equivalent semantically self-sufficient proposition. These certainly include all carefully formulated scientific statements. Semantically non-self-sufficient sentences are the basic building blocks of any conversation: especially the sentences including situationally occasional expressions. This is a large class of sentences if we consider that a situationally occasional use happens to almost all descriptive expressions, rather than to ones such as *me, here, now,* which are listed in logic coursebooks. Semantically non-self-sufficient sentences also include ellipses, but not all of them. A sentence *ja pójdę górą a ty doliną* [I will go uphill and you'll go down a valley] is indeed non-self-sufficient but the reason is not that a predicate has been omitted in its second part as it is acceptable in terms of the rules of grammar to omit sentence parts that repeat further on within the sentence. The sentence *wieloryby są ssakami* [whales are mammals] is semantically non-self-sufficient even though anyone will guess that it refers to all whales, but in selecting such an interpretation they will refer to the elementary biological information that species are contained in classes. When hearing a sentence *wieloryby są złośliwe* [whales are malicious] the addressee would be less determined in their selection of interpretation. Replacing one expression with another seems to be a universal test, allowing us to determine whether in interpreting a sentence we appeal to the knowledge we have. A sentence like *Platon pisał swe dialogi w ostrej polemice z uczniami Sokratesa i postać mistrza służyła mu jako autorytet do poparcia własnego stanowiska* [Plato would write his dialogues in a sharp argument with Socrates's disciples and the personage of the master was used by him as an authority to support his position] is understood only because Socrates was Plato's master. Upon the replacement of the word *mistrz* [master] with the word *wieszcz* [prophet], the sentence becomes unintelligible.

The concept of a semantically self-sufficient sentence may be difficult to be precisely formulated (like all concepts that refer to a natural language), but in my opinion it does introduce a significant distinction. It may mean the same as what we have in mind in the teaching of logic when we speak of a "sentence in a logical sense." Sentences in a logical sense are characterized as expressions which can be assigned a true/false value. 'True' and 'false' are understood as the properties of expressions (treated as a script) at least relativized to a language rather than the extralinguistic circumstances of communication. If so, then these properties can be sensibly attributed to semantically non-self-sufficient sentences only.

The purpose of this study is to present — as a formalized *PL* language — the reconstruction of a small passage of a natural language comprising some pro-forms. The next chapter is introductory and presents a selective analysis of the system of pro-forms of a natural language, illustrated by examples from Polish. The third chapter discusses the lexicon and the basic concepts in the *PL* syntax. The fourth chapter contains the description of the *QL* language that uses the variables and quantifiers as well as the rules of translating *PL* into *QL*. In the fifth chapter the notion of a true sentence in *PL* is defined. The sixth one concerns the issue of the information resource that can be expressed in the *PL* language, while the last chapter outlines the prospects of the definitional expansion of *PL* with new expressions such as pro-forms.

II

Pro-forms make a highly marked class among the words of a natural language and their uniqueness is of a semantic nature. In syntactic terms, that is, the places they occupy in sentence structure are no different from other expressions; they differ among themselves. Both these observations have long been known to grammarians. Grammarians do not say expressly what the semantic uniqueness of pro-forms is about. Some grammarians have rightly noticed that this peculiarity is about the way in which they refer to their objective correlates, but they describe this very generally and erroneously. This manner can best be characterized as representation in a sense which is applied in variables.

In linguists' opinion, the uniqueness of pro-forms is about their being SUBSTITUTES. The phenomenon of substitution is supposed to be about some expressions occurring vicariously in propositions, in place of expressions of a certain class. On this basis, we distinguish between pronouns (substituting nouns), pro-adjectives, pro-adverbs, etc. The origin of the category of substitution seems to be this: the division — accepted in grammar — of words into parts of speech is constructed along the assumed principle of division that includes the considerations of syntax, inflection and semantics. Semantic considerations require that pro-forms be treated as separate parts of speech; the remaining considerations make us include particular kinds of pro-forms into the category of nouns, adjectives, numerals, etc. In saying that pro-forms are substitutes, of sorts, of certain words representing parts of speech, nothing else is stated on top of their behaving like the part of speech in terms of syntax and inflection. A suggestion, present in linguistic propositions, that substitution is an asymmetrical relationship seems erroneous.

A division of pro-forms along the kinds of expressions they substitute, which has been accepted in grammar, inherits all the disadvantages of a division of words into parts of speech, and thus is of little use for this study. Instead of the category of speech, some concepts of logical syntax will be used here. It will just be assumed that, in a natural language, expressions can be identified that are NAMES OF INDIVIDUALS. It can be said of individual names that they all belong to the same SYNTACTIC CATEGORY. The definition of the idea of syntactic category, for the sake of a natural language, goes beyond the purpose of this study. For the record, as understood in this study, syntactic categories do not fulfill the condition: two expressions belong to the same syntactic category iff they are mutually replaceable in any expression, where the expression remains a sentence. The reason for this is the phenomenon of inflection.

The subject of interest here will only be those pro-forms which, in the language of grammarians, are substitutes for individual names. These pro-forms will be counted as individual names in terms of the syntactic category. It ought not to be inferred, however, that these pro-forms are regarded as names. In counting some pro-forms as individual names we only mean to state that, in the structure of a sentence, these pro-forms take the place earmarked for names of individuals.

Speaking of sentences, we will only consider semantically self-sufficient sentences. Thence, pro-forms such as *ja, ty* [I, you] and their inflectional varieties will not be of interest here even though they are substitutes of individual names. Take the following names of self-sufficient sentences:

(1.1) *Sokates jest filozofem.*
[Socrates is a philosopher.]
(1.2) *Ktoś jest filozofem.*
[*Somebody* is a philosopher.]
(1.3) *Każdy jest filozofem.*
[*Everybody* is a philosopher.]
(2.1) *Sokrates nie jest cyklopem.*
[Socrates is not a cyclops.]
(2.2) *Nikt nie jest cyklopem.*
[*Nobody* is a cyclops.]
3.1) *Sokrates jest nauczycielem Platona.*
[Socrates is Plato's teacher.]
(3.2) *Sokrates jest nauczycielem kogoś.*
[Socrates is somebody's teacher.]
(3.3) *Sokrates jest nauczycielem każdego.*
[Socrates is everybody's teacher.]
(4.1) *Sokrates jest wrogiem Platona.*
[Socrates is Plato's enemy.]
(4.2) *Sokrates nie jest wrogiem nikogo.*
[Socrates is nobody's enemy.]
(5.1) *Jeśli Ksantypa jest żoną Sokratesa, to on jest jej mężem.*
[If Xantippa is Socrates' wife, then he is he husband.]
(5.2) *Jeśli Sokrates jest mężem Ksantypy, to ona jest jego żoną.*
[If Socrates is Xantippa's husband, she is his wife.]

These examples demonstrate that the pro-forms *everybody, somebody, nobody, themselves, he, she* and their inflections are substitutes of individual names. This list is not complete as we include pro-forms that represent people (or people, too). Please note that all personal pro-forms can be counted as individuals, such as in the case of the indefinite pronoun *anybody/whoever*, as per the example:
(6) *Whoever is Socrates' enemy is an enemy of Truth.*
takes a position that is inaccessible to individual names (rather, the whole expression "whoever is Socrates' enemy" belongs to the category of individual names).

Also, note that the substitutes of individual names can be compound expressions made up of a pro-form and a general name such as *każdy, pewien,* *żaden* [every..., some... no . . .] or their inflections, with a general name in the place of the dots. Such expressions should thus be treated as belonging to the syntactic category of individual names. This observation suggests a view that is different from the one that holds in logic concerning the syntax of categorical statements (closer to grammatical concepts) and allows for its considerable simplification.

Among the pro-forms indicated above two kinds can be identified. The first includes *everybody, somebody, nobody* and their declensions. On account of their semantic kinship with quantifiers, they will be called 'quantifying pronouns.' For the others, the term 'reflectory pro-forms' seems an apt description as they always remain in a reflectory relation to other expressions. The idea of a reflectory relationship was introduced by Ajdukiewicz, who described it as follows: "[...] reflectory relationships [. . .] assign a member/part its objective correlate only on account of another member/part of the same sentence. [. . .] A reflectory relationship can be exemplified by the relationship obtaining between a pro-form and the noun it pertains to, and this is what assigns the pronoun its denotation" (Ajdukiewicz 1965: 345).

The expression to which a pronoun remains in the reflectory relationship is called by grammarians its antecedent, with the pronoun itself called the consequent of the expression. In sentence 5.1) it is the name 'Socrates' that is the antecedent of the pronoun 'he' and the antecedent of the pronoun 'she' — Xantippa; in sentence 3.4) 'Socrates' is the antecedent of the pronoun 'himself.' In correctly built statements the relation between the antecedent and the consequent is inverse. The antecedent usually precedes the consequent.

In the examples above, it is the expressions that are not pro-forms and are therefore antecedents of reflectory pro-forms. Such an application of reflectory pro-forms is statistically the most commonplace, but from the logical point of view, it is of little interest. The effect thus attained is purely stylistic (avoidance of repetition). For a logician, the cases where a quantifying pronoun is an antecedent to a reflectory pro-form are much more interesting. Reflectory pro-forms then become the equivalents of bound variables. Using reflectory pro-forms this way considerably expands the number of statements that can be uttered in a natural language. The following sentence is an example of this use of a reflectory pro-form:

(7) *Joanna loves someone and he loves Joanna.*

Which, in the functional calculus, is equivalent to the formula:

(7') $\bigvee_{x}(aRx \wedge xRa)$

Possibilities of using reflectory pro-forms in this role are quite limited, though. The occurrence of the antecedent-consequent relationship is established upon a grammatical rule that has it that the words bound by this relationship agree in gender and number. The gender and number is usually marked by way of an inflectional suffix or article. The number of genders in natural languages spans between 2 and allegedly 30, with the numbers from 2 to 4. The number of reflective pro-forms of various antecedents, occurring in the same sentence must therefore be lower than a small natural number. In Polish, we have three genders in the singular and two in the plural. Consequently, in Polish, five reflectory pro-forms of various antecedents can be used in the same statement. Apparently, then, natural languages are in the same situation as the functional calculus where only n varioform variables have been accepted, and therefore some statements cannot be formed in these languages. This is not so, however. What counteracts these limitations will be, among others, using expressions made up of a pro-form and an ordinal numeral, such as: *ktoś pierwszy* [someone first], *kogoś drugiego* [Acc. someone next/second], *ten pierwszy* [the first one], *ten drugi* [the second one], etc. Using numerals in such contexts has nothing to do with their ordinary meanings. Numerals perform the role of inflectional endings here or, if we take the perspective of formalized languages, the role of numeral indexes to variables.

Now onto the properties of quantifying pro-forms, as per the Polish pronouns *każdy* [everybody], *nikt* [nobody] and *ktoś* [somebody] and their declension forms (the pronouns no and *some/a* will be skipped here as they cannot occur without the accompanying general name). The rules of using quantifying pro-forms are tangled because each of these either cannot occur in every context or do not have the same meaning in every permissible context, that is, generalizing or existential. To record the behavior of the three aforementioned pro-forms in (simple) categorical statements. Two kinds of statements need to be accounted for here — affirmative (without inherent negation) and negative (with inherent negation) — as well as two positions of pro-forms: in the subject and in the object. The meaning is, in a given context, decided depending on what quantifier corresponds to it in a given transcription of the sentence onto the language of functional calculus. This transcription consists in the replacement of a pro-form by a variable and appending a quantifier suited to the sense of the sentence. Also, intrasentential negation is replaced by intersentential negation. Intersentential negation always occurs within the range of a quantifier that corresponds to a pronoun in the nominative, but before the quantifier corresponding to a pronoun in the predicative. Then we state:

The pronoun *każdy* [every(body)] can occur in the subject in affirmative sentences only, but in the predicative it can occur in both affirmative and negative statements. It always has a generalizing sense:

(8.1) *Każdy ceni Arystotelesa.*
[Everybody appreciates Aristotle.]
(8.1') $\bigwedge_x (x \textit{ ceni Arystotelesa})$.
[$\bigwedge_x$(x appreciates Aristotle)]
(8.2) *Arystoteles jest autorytetem dla każdego.*
[Aristotle is an authority for everybody.]
(8.2') $\bigwedge_x$ (*Arystoteles jest autorytetem dla* x).
[$\bigwedge_x$ (Aristotle is an authority for x)]
(8.3) *Arystoteles nie jest autorytetem dla każdego.*
[Aristotle is not an authority for everyone.]
(8.3') $\sim \bigwedge_x$ (*Arystoteles jest autorytetem dla* x).
[$\sim \bigwedge_x$ (Aristotle is not an authority for x)]

The pronoun *nikt* [nobody] can occur in both the subject and predicative but only in negative sentences. In the subject it has a generalizing sense but in the predicative — an existential one. Compare:

(9.1) *Nikt nie jest krezusem.*
[Nobody is a Croesus.]
(9.1') $\bigwedge_x \sim$ (x *jest krezusem*).
[$\bigwedge_x \sim$ (x is a Croesus)]
(9.2) *Aleksander nie boi się nikogo.*
[Alexander is not afraid of anyone.]
(9.2') $\sim \bigvee_x$ *Aleksander boi się* x).
[$\sim \bigvee_x$ Alexander is afraid of x)]

The pronoun *ktoś* [somebody] in categorical propositions always has an existential sense. In the object of negative statements the pronoun *ktoś* is not used:

(10.1) *Ktoś zabił Cezara.*
[Somebody killed Caesar]
(10.1') $\bigvee_x (x\ zabi\l\ Cezara)$.
[$\bigvee_x (x$ killed Caesar)]
(10.2) *Brutus zabił kogoś.*
[Brutus killed somebody]
(10.2') $\bigvee_x (Brutus\ zabi\l\ x)$.
[$\bigvee_x$ (Brutus killed x)]
(10.3) *Ktoś nie zdradził Cezara.*
[Somebody did not betray Caesar]
(10.3') $\bigvee_x \sim (x\ zdradzi\l\ Cezara)$.
[$\bigvee_x \sim (x$ betrayed Caesar)]

By denoting the sense of the pro-form with a sign of the corresponding quantifier, the rules formulated above can be presented in tables. Comparing these, we notice that 1) the three pro-forms under scrutiny complement one another and guarantee that in each of the two identified syntactic positions there can be a pro-form both in a general and existential sense, 2) the pro-forms are not doubled in their roles.

everybody		subject	object
sentences	affirmative	$\bigwedge$	$\bigwedge$
	negative	—	$\bigwedge$

nobody		subject	object
sentences	affirmative	—	—
	negative	$\bigwedge$	$\bigvee$

somebody		subject	object
sentences	affirmative	$\bigvee$	$\bigvee$
	negative	$\bigvee$	—

What was said about the use of quantifying pro-forms in the object concerns both the direct and indirect object and can be generalized to the adverbials of time (being subject to quantification with the pro-forms *zawsze* [always], *nigdy* [never], *kiedyś* [some time]) and place (being subject to quantification by means of the adverbials *wszędzie* [everywhere], *nigdzie* [nowhere], and *gdzieś* [somewhere]). The findings of the investigation conducted here can thus be applied to sentences such as:

Każdy komuś kimś grozi.
[Everybody threatens someone with someone.]
Każdy kiedyś przegrywa.
[Everybody loses some time.]
Ktoś wszędzie ma kogoś.
[Everybody has somebody somewhere.]

Note that establishing the meanings of pro-forms according to what quantifier corresponds to them is, in the case of negative categorical propositions, in a way arbitrary as it depends on what place — in a functional formula — is assigned to negation, and as we know, there are three possibilities here. Supposing we assumed that negation occurred in the range of the quantifier corresponding to the pro-form in the object case rather than before it, we would obtain a different result than that in the tables. It would turn out that it is not the pronoun *nikt* [nobody] but rather the pronoun *everybody* which is ambiguous and has an existential meaning in the object of negative propositions. What is independent from the conventions concerning the place of negation is the fact that one of the three pronouns under scrutiny is ambiguous because the result is obtained with any principle of adequate translation into the language that uses quantifiers.

A systematic description of the principles of using quantifying pronouns in compound sentences will be presented in the construction of a suitable formalized language. Here, only some observations will be presented. These observations concern compound sentences where there is a reflectory relationship between the members/parts of two different arguments of the conjunction, as in the sentence:

(11) *Joanna kocha kogoś i on ją kocha.*
[Joanna loves somebody and he also loves her.]

In such sentences at least one component (the argument of the conjunction) is not a semantically self-sufficient proposition as it contains a reflectory pro-form without an antecedent.

The pronouns 'everybody' and 'nobody' perform the role of the antecedent only in few contexts. Thus we say:

(12) *Nikogo nie potępiamy jeśli jego intencje są dobre.*
[We condemn nobody if their intentions are good.]

(13) *Każdy przyzna nam rację jeśli jego przekonania są zgodne z naszym.*
[Everybody will agree we are right if their convictions are in agreement with ours.]

However, the utterance:

(14) *Jeśli każdy jest geniuszem, to jego dzieci są genialne.*
[If everybody is a genius, his children are ingenious.]

is not a statement from the Polish language because the quantifying pronoun *każdy* [everybody] does not bound the reflectory pro-form *jego* [his] even though the formal condition of agreement in number and gender is fulfilled.

The sense of the pronouns *każdy* [everybody] and *nikt* [nobody] that perform the role of the antecedent is always generalized.

The pronoun *ktoś* [somebody] (and its declension forms) is nearly universal in its role of the antecedent of reflectory pro-forms, but it is ambiguous. In the conjunction 11) it has an existential sense, but in the implication:

(15) *Jeśli Joanna kocha kogoś, to również on ją kocha.*
[If Joanna loves somebody, he also loves her.]

it has a generalizing sense.

As a result, in the sentence:

(16) *Jeśli ktoś (pierwszy) jest pracownikiem, to ktoś (drugi) jest dyrektorem, i ten ktoś (drugi) jest zwierzchnikiem tego kogoś (pierwszego).*
[If someone /the first one/ is an employee, somebody /the other/ is a director and this somebody /the other/ is this somebody's /the first one's/ superior.]

it occurs in two meanings.

There is an opinion that the pronouns such as 'somebody' or 'something' perform the role of free variables in natural languages. This opinion does not seem to be correct. These pronouns surely do not perform the roles of free variables in the contexts in which they have an existential sense since — as we know — existential propositions cannot be uttered by means of free variables. Perhaps, then, this is the case when the pronouns have a generalizing sense?

III

In this chapter and the next one a simple formalized language will be presented featuring symbols that have the syntactic and semantic properties of pro-forms. This language — hereinafter referred to as *PL* — can be considered a rational reconstruction of the system of natural language pro-forms that are substitutes of individual names.

Let it be established that the lexicon of *PL* contains a countable string of individual constants:

$a_1, a_2, a_3, \ldots, a_i, \ldots$

Every individual constant is a creation made up of the expression a_i and the $n(n = 0, 1, 2 \ldots)$ index, which is a counterpart of an inflectional suffix and has the form of a_i^n. Four more pro-forms belong to the same syntactic category as individual constants: three quantifying ones and one of a reflectory kind. These are expressions of the following form: $\cap$(general pro-form), $\cup$(particular pro-form), $\frown^k$ a general or particular pro-form, depending on context) ($k = 1, 2, \ldots$) and O^n(reflectory pro-form) ($n = 0, 1, 2, \ldots$).

The choice of the remaining syntactic categories of descriptive terms is largely arbitrary. It has been decided to include a finite number of general names in the *PL* language:

$N_1, N_2, N_3, \ldots, N_r$

as well as the same sequence of relative one-argument predicatives (relatives)

$R_1, R_2, R_3, \ldots, R_s$.

(It would be possible to incorporate in PL transitive and intransitive verbs with the same result.)

The logical constants of *PL* are — on top of pro-forms — the symbols *est* and $\bar{est}$ (read as is and is not) as well as the symbols $\sim, \rightarrow, \wedge, \vee, \leftrightarrow$ which are the signs of intersentential negation, implication, conjunction, alternative and equivalence. In *PL* we use parentheses in accordance with the propositional calculus conventions accepted in Polish.

We will call a *PL* "expression" any finite string of symbols belonging to the *PL* lexicon. A "elementary expression" will be any sequence of symbols created in accordance with one of the following four schemes:

α *est* N_k, α $\bar{est}$ N_k, α *est* $R_1\beta$, α $\bar{est}$ $R_1\beta$,

by way of substituting the syntactic variables α and β any symbols belonging to the syntactic category of individual names (individual constants or pro-forms) and appropriate values in place of the numeral indexes k and l. (It is obvious that not all *PL* expressions make sense in this language and not every *PL* elementary expression is a proposition of this language). The places which the symbol α takes in the above schemes is referred to as "in the subject;" the places which the symbol β takes in the above schemes is referred to as "in the predicative."

The definition of a *PL* proposition is the following inductive definition:

(D1)

1. the *PL* elementary expressions — $a_i^0 est\ N_k$, $\alpha\ \bar{est}\ N_k$, $\alpha\ est\ R_k\ a_j^0$, $\alpha\ \bar{est}\ R_k\ a_j^0$ — are *PL* propositions.

2. if the expressions φ and ψ are *PL* propositions, then the expressions $\sim \varphi,\ \varphi \rightarrow \psi, \varphi \wedge \psi,\ \varphi \vee \psi,\ \varphi \leftrightarrow \psi$are *PL* propositions.

3. If the expression φ is a *PL* proposition, then the expressions $\varphi[a_i^0/\cap]$ and $\varphi[a_i^0/\cup]$ are *PL* propositions.

4. If the expression φ is a *PL* proposition, then the expression $\varphi[a_i^0/O^0]$ is a *PL* proposition providing every elementary expression that is a passage from the expression φ and contains the constant a_i^0contains in the place of the subject one of the expressions $a_j^0(j \neq i)$, $\cap$, $\cup$or O^n (n $\neq$ 0).

5. If φ is a *PL* proposition, then the expression $\varphi[a_i^0/a_j^0,\ O^0]$ and the expression $\varphi[a_i^0/\smile^n,\ O^n]$ $(n \neq 0)$ are *PL* propositions providing the expression φ does not contain the expression $\smile^m$ or a_k^m where there is any k and $m = n$.

One explanation: the abbreviation $\varphi[\alpha/\beta]$ means an expression which is formed from the expression φ by way of replacing in it the expression α by the expression β. The abbreviation $\varphi[\alpha/\beta,\ \gamma]$ signifies an expression that forms from the expression φ by replacing in it the expression α by the expression β in the first position and by the expression γ elsewhere. We assume that 1) (when α does not occur in φ, then $\varphi[\alpha/\beta] = \varphi$; 2) when α does not occur in φ in at least two positions, then $\varphi[\alpha/\beta,\ \gamma] = \varphi$.

A string of notions will now be introduced that will describe the structure of *PL* and some properties of expressions characteristic for *PL* will be demonstrated, and so will the way of reading those, which is in fact a translation into a natural language (Polish).

A *PL* ”categorical proposition” is a proposition that is an elementary proposition of this language. Each sentence of *PL* is either a categorical proposition or a molecular proposition built of elementary expressions by means of conjunctions (including the negation sign $\sim$). Passages that are arguments of a conjunction are called the COMPONENTS of the proposition. The components that are elementary expressions are called ELEMENTARY COMPONENTS. If an elementary component contains the expression *est* it is called AFFIRMATIVE, if it contains $e\bar{s}t$ — a negative one. These terms are also applied to categorical statements. *PL* is characterized by the fact that, like in a natural language, the components of propositions are not always propositions.

The quantifying pro-forms $\cap$ and $\cup$ can — along with the definition (D1) — occur in any elementary statement component both in the subject and in the predicative. Their use is in no way limited apart from the aforementioned syntactic position.

The reflectory pro-form O^0 can occur in the predicative only and only in such an elementary component which has one of the following in the subject: a_i^0, $\cap$, $\cup$or O^n (n $\neq 0$).

The reflectory pronoun O^n (n $\neq 0$) can occur both in the subject and in the predicative but if and only if in the part preceding the given occurrence of the pro-form O^n there occurs at least one occurrence of the constant a_i^m or the quantifying pro-form $\overset{\frown}{\smile}^{m}$ and $m = n$.

The occurrence of the quantifying pro-form $\overset{\frown}{\smile}^{n}$ or the constant a_i^n can occur both in the subject and the predicative but if and only if the part of the proposition following it contains at least one occurrence of the reflectory pro-form O^m and m = n.

The inflection index 0 (zero) is only used when such an index is actually redundant and therefore it will be overlooked.

Note also the condition contained in 5) of the D1 definition: the occurrence of the pro-form $\overset{\frown}{\smile}^{n}$ or the constant a_i^n cannot find themselves in the range of another occurrence of the pro-form or a constant of the same inflectional index. (The expression:

$$\overset{\frown}{\smile}^{1}\ est\ N_k \wedge \overset{\frown}{\smile}^{1}\ est\ R_1 O^1 \rightarrow\ O^1\ est\ N_i$$

is not a proposition from *PL*). This restriction is justified in this way: the pro-from $\overset{\frown}{\smile}^{n}$ is devised as the antecedent of reflectory pro-forms. Its rejection would make some occurrences of the pro-form $\overset{\frown}{\smile}^{n}$ remain without the consequents and that would greatly complicate the reflectory relationships between *PL* expressions.

A string of statements and detailed schemes of *PL* will now be presented as well as the way we should read them. To facilitate the reading , the letters N and M will mostly be used as general names, while R and S will stand for relatives and a and b as individuals. This will stray a little from the *PL* lexicon.

All categoral propositions of *PL* can be obtained from a small pool of schemata, presented below:

(1.1) $a\ est\ N$ a is N
(1.2) $a\ \bar{est}\ N$ a is not N
(2.1) $a\ est\ R\ b$ a is the R of b
(2.2) $a\ \bar{est}\ R\ b$ a is not the R of b
(3.1) $\cap\ est\ N$ everybody is N
(3.2) $\cap\ \bar{est}N$ nobody is N
(3.3) $\cup est\ N$ somebody is N
(3.4) $\cup \bar{est}N$ somebody is not N
(4.1) $\cap est\ R\ a$ everybody is the R of a
(4.2) $\cap \bar{est}R\ a$ nobody is the R of a
(4.3) $\cup est\ R\ a$ somebody is the R of a
(4.4) $\cup \bar{est}R\ a$ somebody is not the R of a
(5.1) $a\ est\ R \cap$ a is the R of everybody
(5.2) $a\ \bar{est}\ R \cap$ a is not the R of everybody
(5.3) $a\ est\ R \cup$ a is the R of somebody
(5.4) $a\ \bar{est}\ R \cup$ a is the R of nobody
(6.1) $\cap\ est\ R \cap$ everybody is the R of everybody
6.2) $\cap \bar{est}R \cap$ nobody is the R of everybody
6.3) $\cap\ est\ R \cup$ everybody is the R of somebody
6.4) $\cap \bar{est}R \cup$ nobody is the R of anybody
6.5) $\cup est\ R \cap$ somebody is the R of everybody
6.6) $\cup \bar{est}R \cap$ somebody is not the R of everybody
(6.7) $\cup est\ R \cup$ somebody is the R of somebody
(6.8) $\cup \bar{est}R \cup$ somebody is not the R of anybody
(7.1) $a\ est\ R\ O$ a is the R of themselves
(7.2) $a\ \bar{est}\ R\ O$ a is not the R of themselves
(8.1) $\cap est\ R\ O$ everybody is the R of themselves
(8.2) $\cap \bar{est}R\ O$ nobody is the R of themselves
(8.3) $\cup est\ R\ O$ somebody is the R of themselves

(8.4) $\cup \bar{est} R\ O$ somebody is not the R of themselves

On top of those, elementary expressions like the following are categorical statements:

(9.1) $\smile^n$ *est* $R\ O^n$ and (9.2) $\smile^n$ $\bar{est}$ $R\ O^n$

which are read: somebody is (is not) the R of themselves (their own R), and thus just like the in 8.3) and 8.4). It is the only case where the reflectory pro-form O^n (n $\neq$ 0) is read: itself (of itself).

The way of reading pro-forms $\cap$, $\cup$, O in compound sentences is the same as in categorical propositions. The quantifying pro-form $\smile^n$ is read: in the place of the subject — *ktoś* [somebody;] in the predicative — *kogoś* [(of/to) somebody]. If needed, we can add an ordinal numeral *ktoś n-ty*, *kogoś n-tego* [n'th somebody, (of/to) nth somebody]. The reflectory pro-form O^n (n $\neq$ 0) in some cases can be read: *on* [he] or *jego* [his]; usually it is: t*en (ktoś) n-ty* [this n'th (somebody) or *tego (kogoś) n-tego* [(to/of) this nth somebody.

When the antecedent of the pro-form O^n (n $\neq$ 0) is an individual constant, a *PL* proposition often does not lend itself to reading, as in the following

(10) $a_1^1 est\ R_1\ a_2^2 \rightarrow\ O^2\ est\ R_2 O^1$,

even though its strict equivalent is the proposition:

Jeśli Ksantypa jest żoną Sokratesa to on jest jej mężem
[If Xantippa is Socrates' wife, he is her husband].

So is the case with the sentence:

(11) $a_1^1 est\ R_1 \smile^2\ \wedge\ O^2\ est\ R_1 O^1$

and with a sentence, similar in structure,

Joanna kocha kogoś i on ją kocha.
[Joanna loves somebody and he also loves her]

But we read this without difficulty:

(12) $\overset{\frown}{\smile}^{1}$ *est* $R\overset{\frown}{\smile}^{2} \wedge O^{1}$*est* $N \to O^{2}$ *est* M,

and that is:

Jeśli ktoś pierwszy jest R-em kogoś drugiego i ten (ktoś) pierwszy jest N-em, to ten (ktoś) drugi jest M-em
[If the first someone is the R of someone (other) and this first (someone) is N, then the (other) someone is M].

More examples of compound sentences of *PL* will be put forward when discussing the issue of the range of quantifying pro-forms.

The notion of the range of the quantifying pro-form is indispensable for the semantic description of the *PL* language and for the precise definition of the notion of the antecedent of a reflectory pro-form. The issue of the range of the quantifying pro-form in the sentence φ is about delimiting such a passage from the proposition φ which, upon adequate translation (due to the interpretation of a natural language determined by the way of its reading), and one possibly as faithful as possible, of the proposition φ into a language using variables and quantifiers, would become the range of the respective quantifier. The issue of range of a quantifying pro-form remains somewhat similar to the notion of quantifier range, but it also differs in significant ways. Trying to maintain the principle that the range of a quantifying pro-form in the proposition φ is always a continuous excerpt of the proposition φ which forms its part, we are forced to come to terms with the fact that two different quantifying pro-forms can have the same range. Another peculiarity of pro-forms is the phenomenon of extending the range of the pro-form by another pro-form. This will be discussed further on.

Let us first describe the range of the quantifying pro-form $\overset{\frown}{\smile}^{n}$. We will use the notion of the component of the nth order of the proposition φ, which can be inductively defined in this way:

The proposition φ is a component of the 0 order of the proposition φ.

The arguments of the main conjunction in the proposition φ are 1^{st} order components of the proposition φ.

The arguments of the main conjunction in the n'th order component of the proposition φ are n + 1^{th} order components of the proposition φ.

(D 2) The range of the given occurrence of the pro-form $\overset{\frown}{\smile}^{n}$ in the proposition φ of the *PL* language is the HIGHEST IN ORDER from among the components of the sentence φ, which:

1. contain the occurrence of the pro-form $\overset{\frown}{\smile}^{n}$

and

2. contain each occurrence of the pro-form O^m if $m = n$ and this occurrence a) takes place in the sentence part that follows the occurrence of the pro-form $\frown^n$ and b) there is no occurrence of the pro-form $\frown^n$ or the constant a_j^k where $k \neq n$ between it and the given occurrence of O^n.

The definition of the range of the individual constant a_j^n (n = 0) has the very same wording.

A set of examples of *PL* propositions will now be put forward where the ranges of the occurrences of the pro-forms $\frown^n$ are marked. Each sentence of *PL* is accompanied by its translation into a natural language and a translation of both into a language using variables and quantifiers. The comparison of the three statements confirms the adequacy of the range definitions proposed here.

$$(13)\quad \underbrace{\frown^1 \mathit{est}\ R \frown^2 \to \underbrace{O^1 \mathit{est}\ R \frown^3 \wedge O^3 \mathit{est}\ R\ O^2}_{\frown^3}}_{\frown^1,\ \frown^2}$$

Jeśli ktoś$_1$ *jest R-em kogoś*$_2$*, to ten*$_1$ *jest R-em kogoś*$_2$ *i ten*$_3$ *jest R-em tego*$_2$*.* [If somebody$_1$ is the *R* of somebody$_2$, then this one$_1$ is the *R* of somebody$_2$ and this somebody$_3$ is the *R* of this one$_2$].
(For an abbreviation of the formula, arithmetic symbols are used instead of numerals).

$$\bigwedge_{x_1}\bigwedge_{x_2} est\ R\ x_2 \to \bigwedge_{x_3}(x_1\ est\ R\ x_3 \wedge x_3\ est\ R\ x_2)]$$

This example also demonstrates that the pro-form $\frown^n$ corresponds to the general quantifier, when its range is implication (so is the case with an alternative and equivalence), but the particular quantifier — when its range is conjunction (or an elementary expression). This principle is in place in Polish also when regarding the pronouns *ktoś* [somebody] and *coś* [something]. Negation is never the range of the pro-form $\frown^n$, which follows from the definition (D1).

$$(14)\quad \underbrace{\underbrace{(a^2\ est\ R\ \overset{\frown}{\smile}{}^1 \to O^1\ est\ R\ O^2)}_{\overset{\frown}{\smile}{}^1} \to O^2\ est\ N}_{a^2}$$

(cannot be read literally)

$$\bigwedge_{x_1} est\ R\ x_1 \rightarrow\ x_1\ est\ R\ a) \rightarrow\ a\ est\ N$$

$$(15)\quad \underbrace{\overset{\frown}{\smile}{}^1\ est\ N \wedge O^1\ \overline{est}\ R\ O \to \sim \underbrace{(\overset{\frown}{\smile}{}^2\ est\ N \to O^1\ est\ R\ O^2)}_{\overset{\frown}{\smile}{}^2}}_{\overset{\frown}{\smile}{}^1}$$

Jeśli ktoś$_1$ *jest N-em i (on) nie jest swoim R-em, to nieprawda, że jeśli ktoś*$_2$ *jest N-em to ten*$_1$ *jest R-em tego*$_2$.
[If somebody$_1$ is N and (he) is not his R, then it is not true that if somebody$_2$ is N then this one$_1$ is the R of this one$_2$].

$$\bigwedge_{x_1}[x_1 est\ N \wedge x_1\ est\ R\ x_1 \rightarrow\ \sim\ \bigwedge_{x_2}(x_2\ est\ N \rightarrow\ x_1\ est\ R\ x_2)]$$

(16) $\overset{\frown}{\smile}{}^1\ \ est\ R\ O^1 \wedge \overset{\frown}{\smile}{}^1\ \ e\bar{s}t R\ O^1$
Ktoś jest swoim R-em i ktoś nie jest swoim R-em. [somebody is their own R and somebody is not their own R].

$$(17)\quad \underbrace{\underbrace{\overset{\frown}{\smile}{}^1\ est\ R\ \overset{\frown}{\smile}{}^2 \wedge O^2\ est\ R\ \overset{\frown}{\smile}{}^3}_{\overset{\frown}{\smile}{}^2} \to O^1\ est\ R\ O^3}_{\overset{\frown}{\smile}{}^1,\ \overset{\frown}{\smile}{}^3}$$

Jeśli ktoś$_1$*, jest R-em kogoś*$_2$ *i ten*$_2$ *jest R-em kogoś*$_3$*, to ten*$_1$ *jest R-em tego*$_3$.
[If someone$_1$ is the R of somebody$_2$ and this one$_2$ is the R of somebody$_3$, then this one$_1$ is the R of this one$_3$].

$$\bigwedge_{x_1}\bigwedge_{x_3}[\bigvee_{x_1}(x_2 est\ R\ x_2 \wedge x_2\ est\ R\ x_3) \rightarrow\ x_1\ est\ R\ x_3]$$

The sentences (17) state that the relationship marked as the relative R is transitory).

The range of the quantifying pro-forms $\cap$ and $\cup$ in any *PL* statement is governed by the following 3 rules:

(R1) If a given occurrence of the quantifying pro-form $\cap$ (or $\cup$) occurs in the predicative of the elementary component of the proposition φ of the language *PL*, then this elementary component is its range in the proposition φ.

The rule can be illustrated by using examples. In order to distinguish between the pro-forms $\cup$ and $\frown^n$ the first one will be read — contrary to the Polish style — as *a/some* (*of a/some*).

$$(18)\quad \underbrace{\underbrace{\frown^1 \; est\ R\ \cap}_{\cap} \to O^1\ est\ R\ O}_{\frown^1}$$

Jeśli ktoś jest R-em każdego to ten ktoś jest swoim R-em.
[If somebody is the *R* of everybody, then this someone is their own *R*],

$$\bigwedge_{x_1}[\bigwedge_{x_2} est\ R\ x) \to\ x_1\ est\ R\ x_1]$$

$$(19)\quad \underbrace{\frown^1 \; est\ N \to \underbrace{O^1\ est\ R\ \cup}_{\cup}}_{\frown^1}$$

Jeśli ktoś jest N-em, to (on) jest R-em pewnego
[If someone is *N*, then they are some's *R*].

$$\bigwedge_{x_1}[est\ N \to\ \bigvee_{x}\ est\ R\ x)]$$

(R 2) If a given occurrence of the quantifying pro-form $\cap$ (or $\cup$) occurs in the subject of an elementary component of the proposition φ of the *PL* language, there is no pro-form $\frown^n$ in the predicative of this elementary component, and the range of this occurrence of the pro-form $\cap$ (or $\cup$) in the proposition φ is this elementary component.

This rule is exemplified as follows:

$$(20)\quad \underbrace{\underbrace{\overset{\frown}{\smile}{}^{1}\ est\ R\ \overset{\frown}{\smile}{}^{2} \wedge \underbrace{\cap\ est\ R\ O^{1}}_{\cap}}_{\overset{\frown}{\smile}{}^{1}} \rightarrow \underbrace{\cup\ est\ R\ O^{2}}_{\cup}}_{\overset{\frown}{\smile}{}^{2}}$$

Jeśli ktoś$_1$ *jest R-em kogoś*$_2$ *i każdy jest R-em tego*$_1$*, to pewien jest R-em tego*$_2$
[If someone$_1$ is the R of someone$_2$ and everyone is the R of this one$_1$, then some is the R of this one$_2$].

$$\bigwedge_{x_2}\{\bigvee_{x_1}[x_1\ est\ R\ x_2 \wedge \bigwedge_{x}(x =\ est\ R\ x_2)] \rightarrow \bigvee_{x_1}(x\ est\ R\ x_2)\}$$

(This example illustrates also how much simpler the structure of the language *PL* can be than the language of the corresponding propositions from the language of the functional calculus).
(R 3) If a given occurrence of the quantifying pro-form $\cap$ (or $\cup$) occurs in the subject of the elementary component of the proposition φ of *PL*, and in the predicative of this elementary component there occurs an occurrence of the pro-form $\overset{\frown}{\smile}{}^{n}$, then the range of the occurrence of the pro-form $\cap$ (or $\cup$) in the proposition φ is the component of the proposition φ which is the range of the occurrence of the pro-form $\overset{\frown}{\smile}{}^{n}$ mentioned.

The rule concerns the phenomenon of extending the range of some quantifying pro-forms by another quantifying pro-form. In the *PL* language, the pro-form $\overset{\frown}{\smile}{}^{n}$ extends — under some circumstances — the range of the pro-forms $\cap$ and $\cup$. This rule is in place in Polish, too, which can be seen when trying to arrive at the sense of the colloquial sentence: *Tutaj każdy zna kogoś, a ten ktoś jest co najmniej dyrektorem departamentu* [here everybody knows somebody and this somebody is at least the director of a department]. It can also find a rationale in the reasons of consistency. Note the statement:

a jest R pewnego N-a [*a* is the *R* of some *N*.]

is a shortcut of

a jest R-em kogoś i ten ktoś jest N-em.
[*a* is the *R* of somebody and this somebody is *N*.]

By means of quantifiers, these statements will be formulated identically:

$$\bigvee_{x_1} est\ R\ x_1 \wedge x_1\ est\ N)$$

Then the sentences:

Każdy jest R-em pewnego N-a.
[Everybody is an R of some N]

and

Każdy jest R-em kogoś i ten ktoś jest N-em.
[Everybody is the R of somebody and this somebody is N.]

ought to be formulated:

$$\bigwedge_{x}\bigvee_{x_1}(x est\ R\ x_1 \wedge x_1\ est\ N)$$

According to the rule (R 3) whole sentence 21) is in the range of a pronoun $\cap$.

$$(21)\quad \underbrace{\cap\ est\ R \smile^1 \wedge\ O^1\ est\ N}_{\cap,\ \smile^1}$$

The definition (D2) and the rules (R1), (R2) and (R3) unambiguously define the range of every occurrence of the quantifying pro-form in any proposition of *PL*.

The relation of being the antecedent of a reflectory pronoun is defined by the following conditions:

(A1) The antecedent of a given occurrence of the reflectory pro-form O^n (n $\neq 0$) can only be an occurrence of the pro-form $\smile^n$ or the constant a_i^n.

(A2) A given occurrence of the pro-form $\smile^n$ (or the constant a_i^n) (n $\neq 0$) is the antecedent of a given occurrence of the pro-form O^m (m $\neq 0$) iff a given occurrence of the pro-form O^m occurs in the range of a given occurrence of the pro-form $\smile^n$ (or the constant a_i^n) and m = n.

(A3) The antecedent of a given occurrence of the reflectory pronoun O^0 can only be an occurrence of the pro-form $\cap$, $\cup$, O^n (n $\neq 0$) or the constant a_i^0.

(A4) The antecedent of a given occurrence of the pro-form O^0 is always the occurrence of the expression $\cap$, $\cup$, O^n or a_i^0 which occurs alongside it in the same elementary expression.

IV

In order to obtain a clear interpretation of the semantic interdependencies between the expressions of *PL* a general method of translating *PL* statements into a formalized language using quantifiers and variables will now be demonstrated. This language will be called *QL*. *QL* does not differ significantly from the standard language of the max. 2-argument predicate calculus. The differences consist just in a different spelling and the presence within *QL* of an intrasentential negation, which can of course be treated as a secondary term.

QL will be made up of:
1) the symbols *est* and $\bar{est}$ and the same conjunctions as in *PL*,
2) the same general names and relatives as in *PL*,
3) individual constants the same as in *PL* but without inflection indexes,
4) individual variables: x_1, x_2, x_3, ..., x_n, ...
5) quantifiers: general $\bigwedge$ and particular $\bigvee$,
6) parentheses.

We call expressions from *QL* any finite strings of symbols belonging to the lexicon of the language *QL*.

The concept of *QL* proposition is defined as follows:

(D3)
1. The expressions of *QL*, of the following form:

$a_i\ est\ N_k,\ a_i\ \bar{est}\ N_k,\ a_i\ est\ R_1 a_j,\ a_i\ \bar{est}\ R_1 a_j$

are *QL* sentences.

2. If *QL* expressions φ and ψ are *QL* sentences, then $\backsim\ \varphi$, $\varphi \rightarrow\ \psi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \leftrightarrow\ \psi$ are *QL* sentences.

3. If for some i the expression of *QL*: $\varphi[x_n/a_i]$ is a *QL* proposition and $\varphi[x_n/a_i] \neq \varphi$ then $\bigwedge\limits_{x_n}\varphi$ and $\bigvee\limits_{x_n}\varphi$ are *QL* sentences.

(This way of defining sentences with quantifiers comes from A. Robinson and, what is characteristic is that it does not allow redundant quantifiers or the use of a quantifier that binds the same variable. *PL* sentences have similar properties regarding the application of redundant inflectional indexes and the use of one quantifying pro-form in the range of the other).

Now the concept of the model of *QL* and a true sentence in this model will be introduced.
The model of the *QL* language is any arrangement of:

$$\mathrm{M} = <U;\ X_1,\ ...,\ X_r;\ Y_1,\ ...,\ Y_s>,$$

where U is a not-empty set, $X_1, \ldots, X_r$ — subsets of U — and $Y_1, \ldots, Y_s$ are subsets of the Cartesian multiplication UXU. There is a Δ_m function related to each model M, which subordinates a denotation to each descriptive constant. We assume that the function Δ_m reflects the set of individual constants of the *QL* language onto the set U, that is, that each element of the set U has a name in *QL*. Because we previously accepted that the set of individual constants of the *QL* language — as is the case in *PL* — is countable, we will only speak of *QL* language countable models. (This limitation is not relevant as owing to a known assertion which has it that any proposition is true in a given model if and only if it is true in a countable model, the semantic notions of consequence and tautology, described in the set of countable models, do not differ in range from the notions described in a set of free models). Also, it is assumed of the function Δ_m that $\Delta_m(N_k) = X_k$ and $\Delta_m(R_2) = Y_1$.

Using the abbreviation $Ver_m(QL)$ to denote the set of *QL* propositions that are true in the Mmodel, the set will be characterized by way of the following conditions:

1. a) $\ulcorner a_i\ est\ N_k \urcorner \in\ Ver_m(QL)$ iff $\Delta_m(a_i) \in \Delta_m(N_k)$,
   b) $\ulcorner a_i\ e\bar{s}t\ N_k \urcorner \in\ Ver_m(QL)$ iff $\Delta_m(a_i) \notin \Delta_m(N_k)$,
   c) $\ulcorner a_i\ est\ R_1 a_j \ulcorner \in\ Ver_m(QL)$ iff $< \Delta_m(a_i),\ \Delta_m(a_j)> \in \Delta_m(R_1)$,
   d) $\ulcorner a_i\ e\bar{s}t\ R_1 a_j \urcorner \in\ Ver_m(QL)$ iff $< \Delta_m(a_i),\ \Delta_m(a_j)> \notin \Delta_m(R_1)$.
2. if the expressions φ and ψ are *QL* propositions, then
   a) $\ulcorner \sim \varphi \urcorner \in\ Ver_m(QL)$ iff $\varphi \notin\ Ver_m(QL)$,
   b) $\ulcorner \varphi \rightarrow \psi \urcorner \in\ Ver_m(QL)$ iff $\varphi \notin\ Ver_m(QL)$ or $\psi \in\ Ver_m(QL)$,
   c) $\ulcorner \varphi \wedge \psi \urcorner \in\ Ver_m(QL)$ iff $\varphi \in\ Ver_m(QL)$ and $\psi \in\ Ver_m(QL)$,
   d) $\ulcorner \varphi \vee \psi \urcorner \in\ Ver_m(QL)$ iff $\varphi \in\ Ver_m(QL)$ or $\psi \in\ Ver_m(QL)$,

e) $\ulcorner \varphi \leftrightarrow \psi \urcorner \in \; Ver_m(QL)$ iff $\varphi \in \; Ver_m(QL)$ only when $\psi \in \; Ver_m(QL)$.
3. If the expression $\bigwedge\limits_{x_n}\varphi$ is a QL sentence, then $\ulcorner \bigwedge\limits_{x_n}\varphi \urcorner \in \; Ver_m(QL)$ iff, for any i, $\varphi[x_n/a_i] \in Ver_m(QL)$.
4. If the expression $\bigvee\limits_{x_n}\varphi$ is a QL sentence, then $\ulcorner \bigvee\limits_{x_n}\varphi \urcorner \in \; Ver_m(QL)$ iff, for a certain i, $\varphi[x_n/a_i] \in Ver_m(QL)$.

Now the method of translating PL propositions into QL statements will be described. The rule of translation that will be used here is similar to those which were applied in the preceding chapter. Making use of the fact that QL has intrasentential negation, we try to obtain as literal a translation as possible so as to facilitate the comparison of QL and PL. One of the principles of the translation applied here is keeping intrasentential negation. The tradeoff is that in some cases — in the predicative of elementary negative statements — the general pro-form is rendered with a particular quantifier, with the particular pro-form rendered by a general quantifier.
So, the sentences:

$a_i \; \bar{est} \; R_1 \cap$ and $a_i \; \bar{est} \; R_1 \cup$

are translated into:

$\bigvee\limits_{x_1} (\; a_i \; \bar{est} \; R_1 \; x_1)$ and $\bigwedge\limits_{x_1} (\; a_i \; \bar{est} \; R_1 \; x_1)$

The translation of the sentence φ of the PL language into QL will be obtained by making the following transformations:

1. if the proposition φ contains the occurrence of the constant a_i^k ($k \neq 0$), then we replace it with the constant a_i (the inflectional index crossed out) and substitute the same expression a_i in place of all occurrences of the pro-form O^k which occurred within the range of the given occurrence of the constant a_i^k and in place of all occurrences of the pro-form O^0 tied with them by a reflectory relationship.

2. If the proposition φ contains the quantifying pro-forms $\overset{n}{\frown\!\!\smile}$, $\cap$ or $\cup$, then:

a. we add parentheses that delimit the range of particular occurrences of the quantifying pro-forms.

b. each occurrence of the pro-form $\overset{\frown}{\smile}^{n}$ is replaced by the variable x_{2n} and the same variable is supplied in place of all occurrences of the O^n pro-form that occurred within its range and in place of all occurrences of the pro-form O^0 associated with it by a reflectory relationship.
c. the occurrences of the pro-forms $\cap$ and $\cup$ are substituted, in the order of their occurrence, with the variables x_1, x_2, x_3, ..., x_i, ... If a given occurrence of $\cap$ (or $\cup$) is associated by a reflectory relationship with an occurrence of the pro-form 0^o, then it is replaced by the same variable as the occurrence of the pronoun $\cap$ (or $\cup$).
d. before the parentheses that delimit the range of a given occurrence of a quantifying pronoun a suitable quantifier is added with which the occurrence of pro-form was replaced by.
The quantifiers are written along with the following assignment:

1. The quantifier $\overset{\frown}{\smile}^{n}$ corresponds to $\bigwedge$ if its range is a conjunction or an elementary expression, and, in the remaining cases — $\bigvee$.

2. The pro-form $\cap$ corresponds to $\bigvee$ in the predicative of elementary negative expressions, and $\bigwedge$ in the remaining cases.

3. The pro-form $\cup$ corresponds to $\bigwedge$ in the predicative of elementary negative expressions, and $\bigvee$ in the remaining cases.

e. in case several occurrences of quantifying pro-forms have the same range, the corresponding quantifiers are placed in the order of these occurrences' taking place.

The adequacy of the method of translation presented here versus the semantic relationships that obtain in a natural language is corroborated by the examples provided previously. The rules of substituting pro-forms with variables described in b. and c. have been adopted for the following reasons:

1. so that no equating would take place between the variables corresponding to the various occurrences of quantifying pro-forms in ways which could change the sense of the utterance;

2. so that the quantifier that binds a given variable would not find itself within the range of another quantifier that binds the same variable which would lead beyond the set of *QL* propositions.

V

In this chapter a definition will be presented of a true *PL* proposition. This notion is relativized to a certain model of *PL*. *PL* models do not differ in anything from the previously described *QL* language models. They are also arrangements of the type:

$M = <U; X_1, ..., X_r; Y_1, ..., Y_s>$

The denotation function related to a given model becomes transformed only in the event of individual constants. We assume here that the function Δ_m reflects the set of constants of the type a_i^0 onto the set U and that for any $k \neq 0$, $\Delta_m(a_i^k) = \Delta_m(a_i^0)$. Like in *QL*, only countable models are considered.

In order to achieve the biggest transparency of definition, the reflectory pro-form O^0 is eliminated from *PL*, whose role in *PL* is insignificant. Notably, the pro-forms $\cap$ and $\cup$ are not indispensable as any proposition of *PL* where these occur may be replaced by a logically equivalent proposition that only includes the pro-forms $\smile^n$ and O^n $(n \neq 0)$. This is argued for in the next chapter.

If the pro-forms $\cap$ and $\cup$ were treated as secondary terms, a definition of a true *PL* proposition would be greatly simplified. This simplification can also be arrived at by the intrasentential negation being counted as a secondary term.

For a greater clarity of the definition the following symbolic acronyms will be used:

Pr (φ) instead of: φ is a *PL* proposition,
El (φ) instead of: φ is an elementary expression,
ElA (φ) instead of φ is an elementary affirmative expression,
ElN (φ) instead of: φ is an elementary negative expression,
C (φ) instead of: φ is conjunction,
A (φ) instead of: φ is alternative,
I (φ) instead of: φ is implication,
Eq (φ) instead of: φ is equivalence,
$a_i^n|\varphi$ instead of: φ is the range of an occurrence of a certain constant a_i^n $(n \neq 0)$,
$1 \smile^n |\varphi$ instead of: φ is the range of an occurrence of a certain pro-form $\smile^n$ and this occurrence precedes all occurrences of quantifying pro-forms having the range φ.

In the case of the pro-forms $\cap$ and $\cup$ we differentiate their position in elementary components, writing:

$1 \cap_{pd}|\varphi$ instead of: φ is the range of some occurrence of the pro-form $\cap$ and 1) this occurrence precedes all occurrences of the quantifying pro-forms that have the range φ and 2) this occurrence comes in the position of subject,

$1 \cap_{or}|\varphi$ instead of: φ is the range of some occurrence of the pro-form $\cap$ and 1) this occurrence precedes all occurrences of quantifying pro-forms having the range φ and 2) this occurrence comes in the predicative.

The symbol $\varphi[\alpha/\beta]$ signifies the result of SUBSTITUTING the expression β for the expression α in the expression φ, while the symbol $\varphi[\alpha,\beta/\gamma]$ — the result of the simultaneous SUBSTITUTING of α and β by γ in the expression φ. The symbol $\varphi[\alpha_1\|\beta]$ signifies the result of the REPLACEMENT in the expression φ of the first occurrence of the expression α with the expression β.

The set of true *PL* sentences in the model *M*is designated by the symbol Ver_m (PL).

Using the designations adopted, the set Ver_m (PL) can be described unambiguously by the following conditions:

(1.1) $\ulcorner a_i^0$ *est* $N_k\urcorner \in Ver_m(PL)$ iff $\Delta_m(a_i^0) \in \Delta_m(N_k)$,
(1.2) $\ulcorner a_i^0$ $e\bar{s}t$ $N_k\urcorner \in Ver_m(PL)$ iff $\Delta_m(a_i^0) \notin \Delta_m(N_k)$,
(1.3) $\ulcorner a_i^0$ *est* $R_1 a_j^0\urcorner \in Ver_m(PL)$ iff $< \Delta_m(a_i^0), \Delta_m(a_j^0)> \in \Delta_m(R_1)$,
(1.4) $\ulcorner a_i^0$ $e\bar{s}t$ $R_1 a_j^0\urcorner \in Ver_m(QL)$ iff $< \Delta_m(a_i^0), \Delta_m(a_j^0)> \notin \Delta_m(R_1)$.
(2.1) If Pr (φ), then $\ulcorner\backsim \varphi\urcorner \in Ver_m(PL)$ iff $\varphi \notin Ver_m(PL)$,
(2.2) If the sentences Pr (φ) and Pr (ψ), then:
  a. $\ulcorner\varphi \rightarrow \psi\urcorner \in Ver_m(PL)$ iff $\varphi \notin Ver_m(PL)$ or $\psi \in Ver_m(PL)$,
  b. $\ulcorner\varphi \wedge \psi\urcorner \in Ver_m(PL)$ iff $\varphi \in Ver_m(PL)$ and $\psi \in Ver_m(PL)$,
  c. $\ulcorner\varphi \vee \psi\urcorner \in Ver_m(PL)$ iff $\varphi \in Ver_m(PL)$ or $\psi \in Ver_m(PL)$,
  d. $\ulcorner\varphi \leftrightarrow \psi\urcorner \in Ver_m(PL)$ iff $\varphi \in Ver_m(PL)$ only when $\psi \in Ver_m(PL)$.
(3.1) If $Zd(\varphi)$ and $a_i^n\,|\varphi$, then $\varphi \in Ver_m(PL)$ iff $\varphi[a_i^n, O^n/a_i^0] \in Ver_m(PL)$.
(4.1) If $Zd(\varphi)$ and (El (φ) or C (φ)) and $1 \smile^{n}\,|\varphi$, then $\varphi \in Ver_m(PL)$ iff for every i, $\varphi[\smile^{n}, O^n/a_i^0] \in Ver_m(PL)$.
(4.2) If $Zd(\varphi)$ and (I (φ) or A (φ) or Eq (φ)) and $1 \smile^{n}\,|\varphi$, then $\varphi \in Ver_m(PL)$ iff for every i, $\varphi[\smile^{n}, O^n/a_i^0] \in Ver_m(PL)$.
(5.1) If $Zd(\varphi)$ and ($1 \cap_{pd}\,|\varphi$ or $1 \cap_{or}\,|\varphi$ and ElA (φ)), then $\varphi \in Ver_m(PL)$ iff for every i, $\varphi[\cap_1\|a_i^0] \in Ver_m(PL)$.

(5.2) If $Zd(\varphi)$ and ElN (φ) and $1 \cap_{or} |\varphi$, then $\varphi \in$ $Ver_m(PL)$ iff for an i, $\varphi[\cap_1 \| a_i^0] \in Ver_m(PL)$.
(6.1) If $Zd(\varphi)$ and ($1 \cup_{pd} |\varphi$ or $1 \cup_{or} |\varphi$ and ElA (φ)), then $\varphi \in$ $Ver_m(PL)$ iff for an i $\varphi[\cup_1 \| a_i^0] \in Ver_m(PL)$.
(6.2) If $Zd(\varphi)$ and ElN (φ) and $1 \cup_{or} |\varphi$, then $\varphi \in$ $Ver_m(PL)$ iff for every i, $\varphi[\cup_1 \| a_i^0] \in Ver_m(PL)$.

The adequacy of the above description of the set $Ver_m(PL)$ can be stated by comparing it to the description of the set $Ver_m(QL)$ by way of the rules previously indicated concerning translating *PL* propositions into *QL*.

In order to establish that it indicates a necessary and sufficient condition of truthfulness for any *PL* proposition, one needs to appeal to the rules of quantifying pro-forms' range, too.

As we know, by using the concept of a true *PL* proposition (relativized to a given model), the function of logical consequence can be defined for the *PL* language. Only the logical calculus of *PL* expressions could provide a direct formal description of this function. It is easy to predict that such a calculus, in its part concerning pro-forms, would considerably differ from standard calculi. It is possible that in some ways such a calculus would be simpler than standard ones as all operations performed on it would come down to transformations that belong to propositional calculus and to the operations of substitution or replacement. At the moment we only have an indirect formal description of the logical consequence function, determined on *PL* expressions: the inference obtaining between *PL* propositions can be settled by having those translated into *QL* and investigating the logical relationships obtaining between the corresponding *QL* propositions.

VI

Comparing *QL* and *PL* a conclusion can be reached that not everything that can be said in *QL* can also be said in *PL* (but the reverse holds true as evidenced by the method of translating *PL* propositions into *QL*, formulated in chapter IV). This conclusion seems to be corroborated by the following observations:

By translating *PL* into *QL* using the method mentioned, we always obtain propositions in which the sequence of quantifiers and the sequence of the occurrence of the corresponding variables is the same. The proposition:

$$\bigvee_{x_1}\bigwedge_{x_2}(x_2 est\ R_1 x_1)$$

cannot be a translation of any *PL* proposition.

Also, the occurrence of the pro-form $\overset{\frown}{\smile}{}^{n}$, whose range is conjunction, always has an existential sense. Hence, the statement:

$$\bigwedge_{x_1}\bigwedge_{x_2}(x_2 est\ R_1 x_2 \wedge x_2\ est\ R_1 x_1)$$

cannot be a translation of any *PL* proposition.

The conclusion which was arrived at in the beginning does not appear relevant though. Based on the fact that the sets of models of *PL* and *QL* are identical, we can use the notion of logical equivalence in reference to propositions each belonging to the other language; in so generalizing the notion of logical equivalence, we assume that:

A sentence φ of the *PL* language and a sentence ψ of *QL* are LOGICALLY EQUIVALENT iff for any Mmodel, $\varphi \in\ Ver_m\ (PL)$ iff $\psi \in\ Ver_m\ (QL)$. The following theorem will now be proved:

(T 1) For any φ of the language *QL* there exists such a sentence ψ of the language *PL* that φ is logically equivalent to ψ.

This theorem can be called one of informational equivalence of *PL* an *QL* as it states that the information resources that can be conveyed by means of *PL* or *QL* are identical.

As we know, every *QL* sentence can be transformed into a logically equivalent proposition of a normal form (with quantifiers at the beginning of the sentence only). In order to prove the theorem (T 1), it is enough to indicate a rule allowing a transformation of a normal *QL* proposition into a logically equivalent *PL* proposition. This rule can be formulated as follows:

1. individual constants a_i is replaced by the constants a_i^0,
2. each occurrence of the variable x_n is replaced by the pro-form O^n,
3. the quantifier $\bigwedge_{x_n}$ is replaced by the expression:

$$(\overset{\frown}{\smile}{}^{n}\ est\ N_k \vee O^n\ e\bar{s}t\ N_k) \rightarrow\ (...$$

4. the quantifier $\bigvee_{x_n}$ is replaced by the expression:

$$(\overset{\frown}{\smile}{}^{n}\ est\ N_k \vee O^n\ e\bar{s}t\ N_k) \wedge (...$$

5. at the end of the formula we append an appropriate number of closing brackets,

6. one can delete brackets that are redundant along with the conventions adopted for *PL*.

This is to illustrate the translation of:

$$\bigvee_{x_1}\bigwedge_{x_2}(x_2 est\ R_1 x_1),$$

into:

$$(\supset^n\ est\ N_k \vee O^1\ e\bar{s}t\ N_k) \wedge (\supset^2\ est\ N_k \vee O^2\ e\bar{s}t\ N_k \rightarrow\ O^2\ est\ R_1 O^1)$$

Further on it will be proved that the way of translating normal *QL* propositions into *PL*, that has been adopted, leads to propositions that are equivalent with the starting sentences.

Let φ signify any normal *QL* proposition and $P_1(\varphi)$ — its translation into *PL*. Let $P_2[P_1(\varphi)]$ signify the translation of the proposition $P_1(\varphi)$ (of *PL*) into *QL* along the rules established in chapter IV. In order to prove that the proposition φ is logically equivalent to $P_1(\varphi)$, it is enough to prove that the sentence $P_1(\varphi)$ is logically equivalent to $P_2[P_1(\varphi)]$ and that the latter is logically equivalent to the proposition φ. The propositions $P_1(\varphi)$ and $P_2[P_1(\varphi)]$ are logically equivalent because the description of the set Ver_m (*PL*) was so selected that the proposition ψ from the language *PL* should belong to Ver_m (*PL*) iff $P_2(\psi)$ belongs to Ver_m (*QL*). It is then enough to prove that the propositions φ and $P_2[P_1(\varphi)]$ are logically equivalent. Both these propositions belong to *QL*, but it is easy to notice that they cannot be identical even when in the sentence $P_2[P_1(\varphi)]$ the indexes at the variables are divided by 2 (which can always be done as in the sentence $P_1(\varphi)$ the quantifying pro-forms $\cap$ and $\cup$ do not occur, so all indexes are even). The expression $P_2[P_1(\varphi)]$ is then different from the expression φ in the presence of tautological formulas of the type:

$$x^n\ est\ N_k \vee x^n\ e\bar{s}t\ N_k.$$

In our example, the proposition $P_2[P_1(\varphi)]$ looks as follows:

$$\bigwedge_{x_1}[(x^1\ est\ N_k \vee x^1\ e\bar{s}t\ N_k) \wedge \bigwedge_{x_2}[(x^2\ est\ N_k \vee x^2\ e\bar{s}t\ N_k \rightarrow\ x^2\ est\ R_1 x_1)].$$

The proposition $P_2[P_1(\varphi)]$ is, however, logically equivalent to the proposition φ because — as can easily be argued — every proposition of the type:

$$\bigwedge_{x_n}[\tau\ (x_n) \rightarrow\ \alpha(x_n)],$$

where $\tau(x_n)$ is a tautological formula, is logically equivalent to the proposition:

$$\bigwedge_{x_n}\alpha\ (x_n)$$

and also every proposition of the type:

$$\bigvee_{x_n}[\tau\ (x_n) \wedge \alpha(x_n)]$$

is logically equivalent to the proposition

$$\bigwedge_{x_n}\alpha(x_n).$$

Note that the *PL* sentences that are translations of a normal form of the language *QL* do not include quantifying pro-forms $\cap$ or $\cup$. We know that for any statement φ of the language *PL* there is a logically equivalent proposition ψ of the language *QL* which, in turn, is logically equivalent to its normal form $\mathrm{N}(\psi)$, and this one — to a *PL* proposition $P_1[N(\psi)]$, which does not contain the pro-forms $\cap$ and $\cup$. It hence follows that:

(T 2) For every *PL* proposition φ there is a logically equivalent proposition φ' of the same language that has no quantifying pro-forms $\cap$ or $\cup$.

The quantifying pro-forms $\cap$ and $\cup$ are thus — given a certain meaning of the phrase — secondary terms on account of the quantifying pro-form $\overset{\frown}{\smile}^{n}$ and the reflectory pro-form O^n.

VII

The comparison of *PL* propositions with expressions from the natural language that are approximate in structure reveals that in a natural language the same can generally be said in another — much shorter — manner. This is illustrated by the three equivalent propositions:

(1.1) $\overset{\frown}{\smile}^{1}$ *est* $N \rightarrow\ O^1$ *est* $R\ \overset{\frown}{\smile}^{2} \wedge O^2$ *est* M.

(1.2) If someone$_1$ is N, then this one$_1$ is someone$_2$'s R, and this one$_2$ is M.
(1.3) Every N is some M's R.

The abbreviation of the utterance is about bringing complex propositions or components of complex propositions to a categorical form by way of using syntactic forms, not yet discussed here, of the type: *every (a, no) N, of every (a, no) N*. We can also abbreviate utterances in a natural language by using conjunctions as name-generating functors and a similar usage of negation. We thus obtain general names and relatives of the type: non-N (non-sentimental), N and M (scholar and administrator), N or M (engineer or technician), R and S (friend and adviser), R or S (maternal uncle or paternal uncle).

There are no reasons why all such syntactic forms should not find their way to *PL*. As the subject matter of this discussion are pro-forms only, we will stop at enriching *PL* in two new syntactic forms: $\cap_N$and $\cup_N$. We take it for granted that in relation to them the same syntactic rules apply as those which govern the use of the pro-forms $\cap$ and $\cup$ (*cf.* the definition of a *PL* proposition {D1} 3, 4). These expressions, just like $\cap$ and $\cup$ are counted into the syntactic category of individual names. The expression $\cap_N$ reads: *(of) every N* or *no N* (in the subject of elementary negative expressions), whereas the expression $\cup_N$ *(of) an N* or *no N* (in the predicative of elementary negative expressions).

It will be demonstrated that the expressions $\cap_N$ and $\cup_N$ are secondary in *PL* on account of the pro-forms $\overset{\frown}{\smile}{}^{n}$ and O^n, by indicating a set of rules that allow the elimination of the expressions $\cap_N$ and $\cup_N$ from any *PL* proposition. These rules have the form of replacement rules. Each of these allows the replacement of φ by ψ in any proposition. The expressions φ and ψ are propositions or components of propositions. In forming the rules, the syntactic variable α will be used which represents any *PL* expressions in individual names category, that is:

$$a_i^n (n = 0, 1, 2, ...), \overset{\frown}{\smile}{}^{n} \ (n = 1, 2, ...), O^n (n = 0, 1, 2, ...), \cap, \cup$$

and $\cap_N$, $\cup_N$. Using the rules is subject to the following two conditions: 1) the expression φ (replacement) occurs in the range of no occurrence of the pro-form $\overset{\frown}{\smile}{}^{n}$ or the constant a_i^n, 2) if α is an expression of a form of a_i^m, $\overset{\frown}{\smile}{}^{m}$ or O^m (m $\neq$ 0), then m $\neq$ n.

For the sake of simplifying the rules discussed here, it is convenient to use in some cases the symbol of reverse implication, which for this purpose is introduced into the vocabulary of the *PL* language. The sign of reverse implication ← is read . . . ., if . . . . . Using the symbol: est/$\overline{est}$ means that the rule concerns both elementary affirmative expressions and negative ones.

These are the rules of the elimination of the expressions $\cap_N$ and $\cup_N$:

(R 1.1) $$\frac{\underset{N}{\cap}\ est/\overline{est}\ M}{\overset{\frown}{\smile}^{n} est\ N \rightarrow O^{n}\ est/\overline{est}\ M}$$

(R 1.2) $$\frac{\underset{N}{\cap}\ est/\overline{est}\ R\ \alpha}{\overset{\frown}{\smile}^{n} est\ N \rightarrow O^{n}\ est/\overline{est}\ R\ \alpha}$$

(R 1.3) $$\frac{\alpha\ est\ R\ \underset{N}{\cap}}{\alpha\ est\ R\ \overset{\frown}{\smile}^{n} \leftarrow O^{n}\ \overline{est}\ N}$$

(R 1.4) $$\frac{\alpha\ \overline{est}\ R\ \underset{N}{\cap}}{\alpha\ \overline{est}\ R\ \overset{\frown}{\smile}^{n} \wedge O^{n}\ est\ N}$$

(R 2.1) $$\frac{\underset{N}{\cup}\ est/\overline{est}\ M}{\overset{\frown}{\smile}^{n} est\ N \wedge O^{n}\ est/\overline{est}\ M}$$

(R 2.2) $$\frac{\underset{N}{\cup}\ est/\overline{est}\ R\ \alpha}{\overset{\frown}{\smile}^{n}\ est\ N \wedge O^{n}\ est/\overline{est}\ R\ \alpha}$$

(R 2.3) $$\frac{\alpha\ est\ R\ \underset{N}{\cup}}{\alpha\ est\ R\ \overset{\frown}{\smile}^{n} \wedge O^{n}\ est\ N}$$

(R 2.4) $$\frac{\alpha\ \overline{est}\ R\ \underset{N}{\cup}}{\alpha\ \overline{est}\ R\ \overset{\frown}{\smile}^{n} \leftarrow O^{n}\ est\ N}$$

The process of the elimination of the expressions $\cap_N$ and $\cup_N$will be illustrated by the example of a *PL* counterpart of (1.3), that is:

(2.1) $\cap_N est\ R\ \cup_M$.

Using rule (R 1.2) we obtain:

(2.2) $\overset{\frown}{\smile}^{1}\quad est\ N \rightarrow\ O^1\ est\ R\ \cup_M$.

Using rule (R 2.3) to the consequent of the expression (2.2), we obtain:

(2.3) $\smile^{1} \quad est\ N \rightarrow \ O^{1}\ est\ R \smile^{2}\ \wedge O^{2}\ est\ M,$

that is, the expression (1.1).

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